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ORIGINAL ARTICLE

Application of an original method to the design of composite beams and comparison with the American Standard NDS 2005

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Abstract: In the elastic field, for beams of rectangular cross-section composed of several vertical layers of materials and linked through horizontally arranged mechanical connectors, it is necessary to know the law of horizontal shear stresses (τ_{zx}) to determine the cross-section and spacing of such connectors. Currently, there are only two methods to solve the problem without taking into account the finite element method: the Empirical Method, without a theoretical basis, and the Rational Method, based on the NDS 2005, which does not take into account the horizontal shear stresses (τ_{zx}). According to numerous authors, both methods lead to conservative results in the spacing of mechanical connectors. A novel method, based on a simplified equation, describes the law of horizontal shear stresses (τ_{zx}) and solves the problem simply and with acceptable accuracy. Two equations catalogued as exact solutions to the stress problem that had never been used for this purpose before have been applied to verify the results. In addition, to validate the new elastic method, information from four-point bending tests performed on specimens of wood beams reinforced with steel plates linked with different connection means, such as bolts, screws, nails and combinations, was used to validate the new elastic method. Fifty-four states were analyzed for flitch beams, varying the magnitude of a uniform load and the span for a simply supported beam. The mechanical connector was adopted as ½ inch for all cases. The Rational and Elastic methods used the NDS 2005 specifications to consider humidity and temperature. The results showed that the spacing of the mechanical connectors according to the Rational Method was conservative since it required a design load perpendicular to the grain. In addition, the New Method made it possible to obtain a larger spacing between mechanical connectors, reducing the number of bolts without affecting the resistance to horizontal shear stresses of the beam. The New Method demonstrated great practical utility and potential to be incorporated into the Allowable Stress Design method of the NDS 2005.

Keywords: horizontal shear stress, composite beam, flitch-beam, elastic

1 Introduction

During the last decades, the construction sector has been one of the main contributors to atmospheric CO2 emissions and climate change, as exposed in the research [1, 2]. [3] emphasizes the importance and relevance of selecting construction products based on their environmental impact. For this purpose, the internationally accepted method of Life Cycle Assessment is used to quantify the

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effects associated with the production and extraction of resources. In a comparative study by [3] of three types of housing made of wood frame, steel frame, and concrete, according to the Ontario Building Code, it was determined that the lowest environmental impact was associated with wood construction. This does not mean that it should always be built in wood, since it has its limitations. Therefore, a valid option is always to combine it with metal and concrete to obtain hybrid elements, as in the case of composite beams such as the flitch-beams, among others. Arguably, this has motivated researchers to focus on the need to develop new sustainable and environmentally friendly construction methods, as mentioned by [4, 5, 6]. Undoubtedly, this has significantly contributed to promoting wood construction as a sustainable alternative to concrete and steel. The research conducted by [7] evaluated different methods of reinforcement of timber beams with steel plates through laboratory tests, emphasizing the great importance of developing design guidelines related to reinforcement methods. On the other hand, [8,9] have conducted studies highlighting the great importance and influence of mechanical connector arrangement and spacing patterns on the flexural properties of composite timber beams reinforced with steel plates. Thus, it was observed that an increase in the density of the arrangement of the mechanical connectors (bolts) slightly increased the flexural stiffness, while, on the contrary, there was a significant reduction in the flexural strength. Despite the existence of numerous research works on the subject of metal-reinforced composite beams, to explore new structural possibilities through the combination of both materials, according to [10], the research is relatively recent and dates back to the 1940s; even some patents have already been registered in the 1920s. Nevertheless, despite being a topic whose antiquity goes back at least as far as 1883, according to [11], currently, only a small group of authors have attempted to develop a classification system to systematize the existing information, through different approaches, from the geometry of the layout to the characteristics of the materials, the means used to connect the materials, and even the objectives pursued. It was not until recent years that a considerable amount of research work on the subject emerged. Structural designers and builders, in the search and exploration of new possibilities, through the combination of different materials such as wood and metal, as in the case of flitch-beams, may have experimented with and used these hybrid structural elements long before their formal publication in 1883, as discussed in [10], referring to the year 1859.

Interestingly, to this day, there are only two widely used methods to justify the section and explain the distribution pattern of the mechanical connectors, both at the supports and along the length of the beam. One is the Empirical Method, and the other is the Rational Method, which will be developed based on the 2005 National Design Specification for Wood Construction. In this research work, a new Elastic Method Analysis will be introduced with a novel and innovative approach, using original and simplified equations given by [20], to determine the pattern and spacing of mechanical connectors in a simple and acceptably accurate way for rectangular cross-section beams of considerable width. As an alternative control procedure to validate the results of the new method, the complex and scarcely diffused solution provided by the Theory of Elasticity will be used. Finally, it will be shown that the results obtained from the application of the Rational Method are extremely conservative concerning the proposed New Method. Furthermore, it has a direct impact on the understanding of the real phenomenon of horizontal shear stresses (τ_{zx}) developed between the vertical planes of contact between layers of materials and in the design practices of composite beams. In addition, the equations used are of great simplicity and easy application, compared to the complex exact solution obtained through the theory of elasticity.

2. Materials and Methods

2.1 Classification of composite beams

This work focuses on the determination of the distribution and spacing pattern of horizontally arranged mechanical connectors in composite beams of rectangular cross-section, of considerable width, consisting of timber planks combined with steel plates. Within this typology, we can cite flitch beams and built-up beams, also referred to [12] and [13] as an option to construct timber trimmer joists, frequently used in timber floors of all domestic buildings, to form a trimmer beam. According to [14], flitch beams exhibit notable advantages such as reduced weight and cost compared to all-metal composite beams. Wood sidings allow builders to use screws or nails to fasten joists or secondary beams, which has a direct impact on reducing overall costs. They are also easy to install, especially in restoration projects, where there are usually aesthetic requirements. Their versatility and resistance compared to their dimensions make flitch beams an interesting option compared to traditional beams.

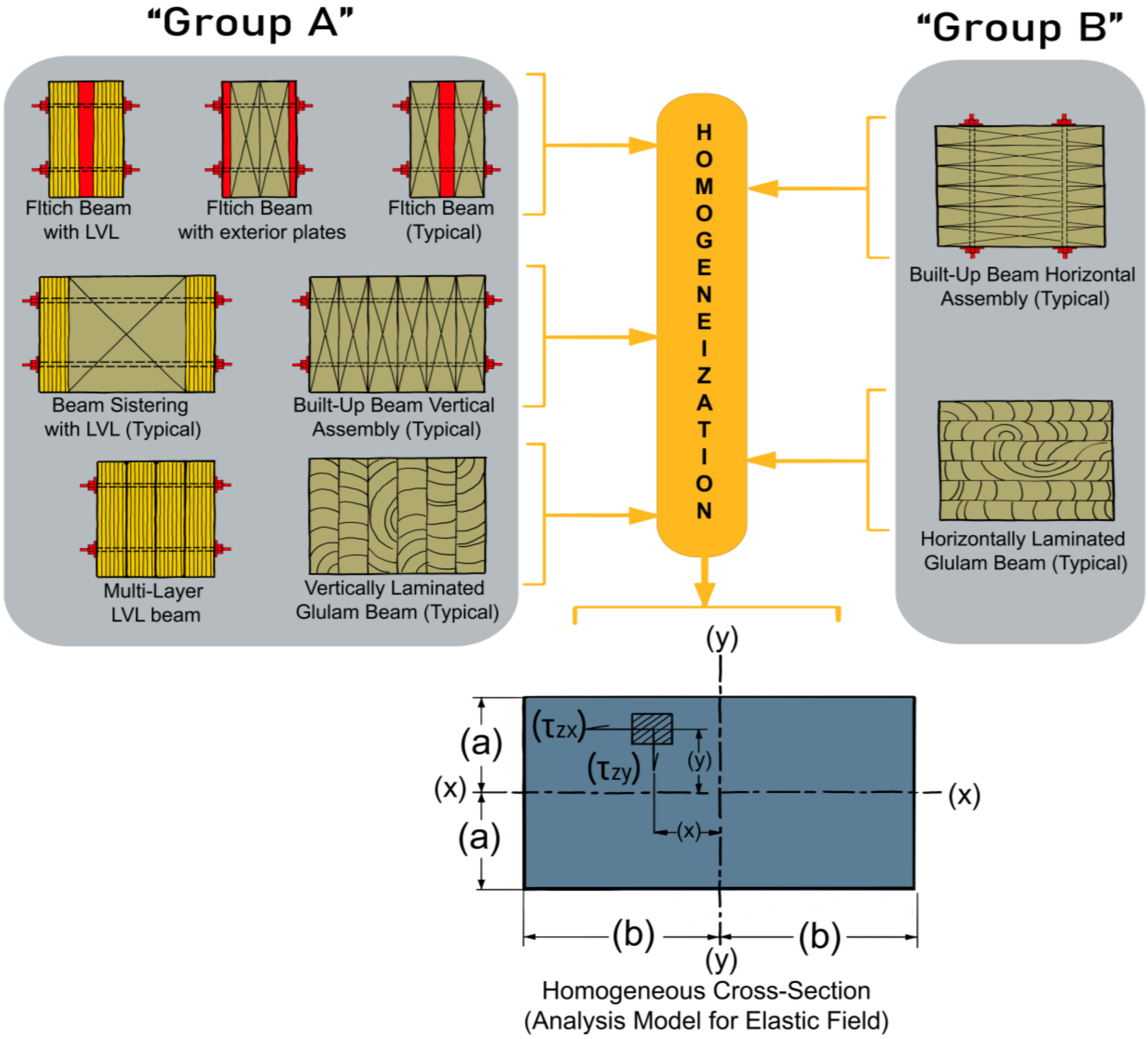

Fig. 1. Classification of composite beam types subjected to homogenization and stress analysis within the elastic field. The composite cross-sections, after homogenization, are assimilated to a rectangular cross-section.

However, among their disadvantages is that their effectiveness is surpassed by other, more modern types of beams, such as glulam beams. In addition, they require a longer assembly time. It is also an essential option for twin beams, either for structural reinforcement due to changes in original load states or due to structural damage from failure or fire, moisture, and insects, among other factors. As pointed out by [15], within the two traditionally used methods, there are numerous design criteria, which vary greatly from one designer to another. According to [16], it is noted that there are different patterns of arrangement of mechanical connectors, some of them based on the observation of structures already built, which have demonstrated satisfactory behavior over the years. [16] points out that, in essence, there are two methods, the Empirical Method and the Rational Method, with some considerations and adaptations according to the criteria of each designer. Curiously, the Rational Method does not consider the horizontal shear stresses (τ_{zx}) generated between the vertical contact planes between the vertical layers of materials. In this research, both methods will be developed. Mainly, the Rational Method is based on the equations and tables given by [17], within the elastic field, using the Allowable Stress Design approach. According to **Fig.1**, the most frequent cases of composite beams of rectangular cross-sections within the suspended timber flooring defined by [12] are presented in two groups. As these are composite cross-sections, it is necessary to homogenize the cross-section to perform the elastic analysis. After such a procedure, all the cases shown in **Fig. 1** lead to the case of a homogeneous beam with a rectangular cross-section of considerable width. Indeed, there are other composite sections with geometries of greater complexity, as shown in Fig. 2; however, through the Collignon-Zhuravsky equation given by Eq. (1), it is possible to estimate the horizontal and vertical shear stresses reasonably. It is also true that very little is known about the law of horizontal shear stresses (τ_{zx}) in composite rectangular sections. For this reason, the study is focused on the case of flitch beams since its solution can be extrapolated and generalized to the rest of the sections of group A in **Fig. 1**.

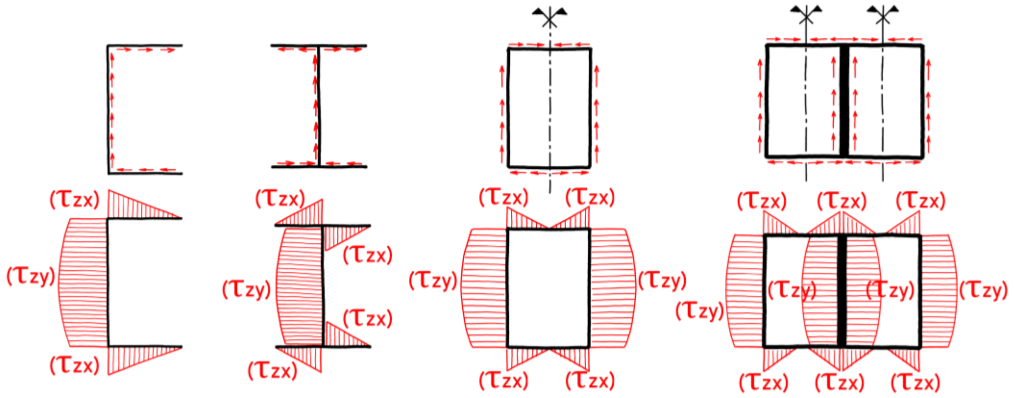

Fig. 2. Shear stress flow in composite beams with complex geometries according to [28,29] and [40].

Another type of composite section, extremely popular among structural designers, is glued laminated timber or glulam beams. Glued laminated timber, or glulam, is the result of stacking boards or lamellas on top of each other, glued together, to form the desired cross-section. It has been used for more than a century as a material with superior performance compared to solid wood. The advantages of glued laminated timber are improved strength and stiffness, freedom to design geometric shapes, and the possibility of using laminations of different quality according to the expected stress levels, improved dimensional accuracy, and shape stability during exposure to humidity. [37]

For the design of glulam beams, there are alternative procedures with greater sophistication and complexity. One of them is the simplified static procedure proposed by [27] for designing composite beams with interlayer sliding. It can be applied to arbitrary boundary and loading conditions to predict the internal strains and stresses. The fully composite bending stiffness (EI_∞) is replaced by the (partially) composite effective bending stiffness (EI_{eff}). The method was verified in an application to a series of simple practical cases, with excellent results that have been verified with exact values.

The vast majority of theoretical models of glued-laminated timber for determining its mechanical performance are based on stochastic analysis through Monte Carlo simulations. On the other hand, other glued-laminated timber models are based on finite element programs through plane stress analysis. An interesting review of modeling techniques for glulam beams is provided by [37]:

- Glulam models for manual calculation: Some proposed models use the so-called section transformation method, with the typical assumptions of conventional beam theory (Bernoulli-Euler). It is a deterministic model, combined with a material with elastic behavior. The main problem is that it is not possible to obtain information about the distribution of the bending resistance and requires the classical Weibull theory. There is also an early empirical model, developed in the 1940s, applicable for glued timber, called the (I_k/I_g) method, mostly used in Canada and the US. The method uses the reduction of the moment of inertia due to the presence of nodes as a way to account for the effects of drag reduction. I_k is the moment of inertia of the nodes and I_g is the moment of inertia of the beam cross-section. However, it does not allow the beam strength distribution to be calculated.
- Glulam models for computer analysis: There are several theoretical models for predicting the bearing capacity of glulam beams in bending. Although the models mentioned have slightly different approaches in terms of mechanical modelling, they use computational simulations based on the principle of dividing the beam into zones and assigning material properties, taking into account their variability.

3. Theory

3.1. Equations of the theory of elasticity for the description of the stress phenomena

When the sections present a considerable width compared to their depth, the value acquired by the horizontal shear stress (τ_{zx}), reaches a relevant magnitude, to the point of exceeding the maximum value of the vertical shear stress (τ_{zy}), obtained through the Elementary Shear Theory, given by Eq. (1). In the case of flitch beams, to take into consideration the action of the horizontal shear stresses (τ_{zx}) developed in the vertical planes of contact between vertical layers of materials, Eq. (2) given by [21] must be used. It is also possible to use a more compact version, given by Eq. [3], according to [22], which, although it differs substantially from Eq. (2), still retains considerable complexity at the time of its application, so it is not of great practicality.

Consequently, as a valid alternative, of acceptable accuracy and great ease of application, comparable to the Zhuravsky-Collignon equation, given by Eq. (1), it is proposed to use the simplified Eqs. (4), (5), and (6) given by [20]. In **Fig. 4**, Hypothesis (A) shows the law of variation of horizontal shear stresses (τ_{zx}), according to [21, 22]. On the other hand, Hypothesis (B), according to [20], assumes a parabolic distribution law, which greatly simplifies the stress calculation process while maintaining a reasonable accuracy in its results. As mentioned by [20], possibly the limited diffusion and complexity of Eqs. (2) and (3) have contributed to marginalizing their application for so many years, especially to consolidate as an answer to the development of solid theoretical support for the Rational Method.

Depending on whether the material layers are arranged vertically or horizontally, the corresponding shear stress is determined (See **Fig. 5**). For example, in the case of vertically assembled layers of materials (**Fig. 5**, Case B), the horizontal shear stress (τ_{zx}) should be used to define the pattern and spacing of horizontally arranged mechanical connectors, according to Eqs. (2), (3), (4), (5), and (6). The distribution law of horizontal shear stresses (τ_{zx}) can be observed in **Fig. 4**.

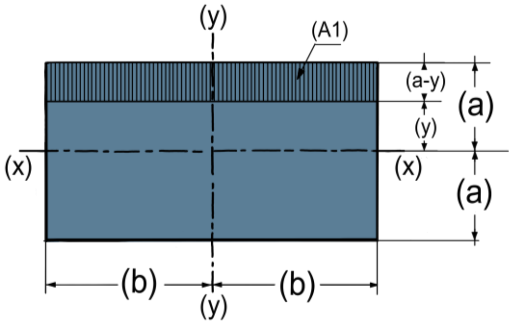

Fig.3. Homogenized cross-section, corresponding to a composite cross-section beam. The geometrical parameters used in equations [1] and [6] are indicated.

$$\tau_{zy} = \tau_{\substack{\text{Collignon} \\ \text{Zhuravsky}}} = \frac{QS^n}{I \times 2b} \tag{1}$$

Where: (τ_{zy}): vertical shear stress (According to [40-43]). For a rectangular cross-section of area "*A*", subjected to a vertical shear "*Q*", the vertical shear stress reaches the value (3*Q*/2*A*) on the neutral axis; (*Q*): vertical shear, in the transverse section considered; (S^n): static moment of the shaded section "A_1", concerning the neutral axis. (See **Fig.3**); (*I*): barycentric moment of inertia of the complete cross-section with respect to the x-x-axis; (2*b*): total width of the beam (See **Fig.3**).

$$\tau_{zx} = \frac{3P}{2A}\left\{\frac{v}{1+v}\frac{16b}{\pi^2 \mathrm{a}}\sum\frac{(-1)^n \sin\frac{(2n+1)\pi y}{2a}}{(2n+1)^2}\left[\frac{x}{b} - \frac{\sin\frac{(2n+1)\pi x}{2a}}{\sin\frac{(2n+1)\pi b}{2a}}\right]\right\} \tag{2}$$

Where: (τ_{zx}): horizontal shear stress (See **Fig.4**, hypothesis A, according to [21]); (*P*): vertical shear, in the transverse section considered; (y): vertical y-coordinate of the considered analysis point; (x): horizontal x-coordinate of the analysis point under consideration; (a): half of the beam depth (See **Figs.3** and **9**); (b): half of the beam width (See **Figs.3** and **9**); (A): cross-sectional area considered $A=(2a)(2b)$; (ν): Poisson's modulus, 0.30 to 0.25 according to [21],[36], and [39]. Adopted ν=0.30; (n): number of terms in the series (at least 10 terms are recommended).

$$\tau_{zx} = \frac{2\nu a^2}{(1+\nu)\pi^2 I_X} \sum_{n=1}^{\infty} \left[\frac{(-1)^n}{n^2} \frac{\sinh \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}}{\cosh \frac{n\pi b}{a}} \right] \tag{3}$$

Where: (τ_{zx}): horizontal shear stress (See **Fig.4**, hypothesis A, according to [22]); (P): vertical shear, in the transverse section considered; (y): vertical y-coordinate of the considered analysis point; (x): horizontal x-coordinate of the analysis point under consideration; (a): half of the beam depth (See **Figs.3** and **9**); (I_x): barycentric moment of inertia of the complete cross-section with respect to the x-x-axis; (ν): Poisson's modulus, 0.30 to 0.25 according to [21],[36], and [39]. Adopted ν=0.30; (n): number of terms in the series (at least 10 terms are recommended).

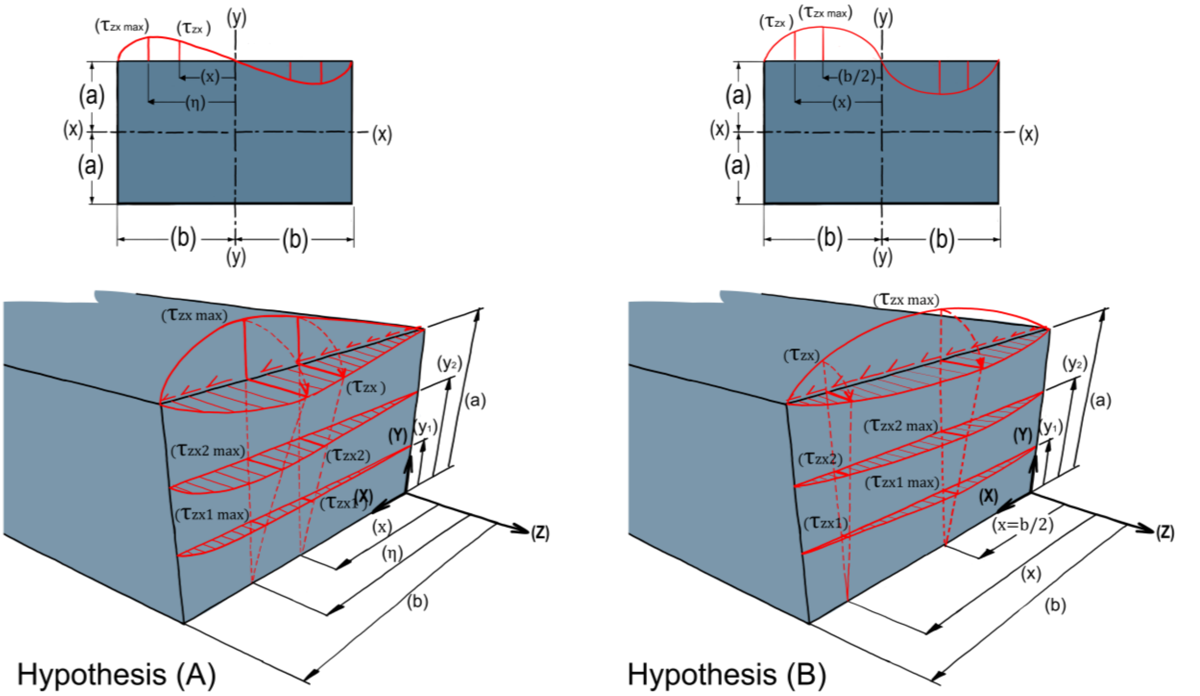

Fig.4. Law of variation of horizontal shear forces, according to the theory of elasticity. Hypothesis (A) corresponds to the exact solution of the theory of elasticity given by Eqs. (2) and (3), according to [21,22] respectively. Hypothesis (B) corresponds to the simplified equation given by Eqs. (5) and (6) according to [20].

$$\tau_{zx\,\text{average}} = \left[\frac{9}{32} \left(\frac{b}{a} \right) \frac{Q}{A} \right] \left(\frac{y}{a} \right) \tag{4}$$

Where: ($\tau_{zx\ average}$): average horizontal shear stress, at the coordinates point ($x=b/2$; y), (See **Fig.4**, hypothesis B, according to [20]); (Q): vertical shear, in the transverse section considered; (a): half of the beam depth (See **Figs.3** and **9**); (b): half of the beam width (See **Figs.3** and **9**); (A): cross-sectional area considered $A=(2a)(2b)$.

$$\tau_{\substack{zx\,\text{maximum} \\ \left(x=\frac{b}{2};y\right)}} = \left[\frac{9}{32} \left(\frac{b}{a} \right) \tau_{\substack{\text{Collignon} \\ \text{Zhuravsky}}} \right] \left(\frac{y}{a} \right) \tag{5}$$

Where: ($\tau_{zx\ maximum;\ (x=\frac{b}{2};y)}$): maximum horizontal shear stress, at the coordinates point (x=b/2; y), (See **Fig.4**, hypothesis B, according to [20]); ($\tau_{zy\ Collignon-Zhuravksy}$): vertical shear stress. For a rectangular cross-section of area "A", subjected to a vertical shear "Q", the shear stress is equal to (3Q/2A) in the neutral axis; (Q): vertical shear, in the transverse section considered; (y): vertical y-coordinate of the considered analysis point; (x): horizontal x-coordinate at point x=b/2; (A): cross-sectional area considered; (a): half of the beam depth (See **Figs.3** and **9**); (b): half of the beam width (See **Figs.3** and **9**).

$$\tau_{\substack{\text{zxjoint}\\(x;y)}} = 6\left[\tau_{\substack{\text{zxaverage}\\(x;y)}}\right]\left(\frac{y}{a}\right)\left[\left(\frac{y}{b}\right)-\left(\frac{x}{b}\right)^2\right] \quad (6)$$

Where: ($\tau_{zx\ joint\ (x;y)}$: horizontal shear stress, at a point of coordinates (x, y), belonging to the vertical plane of contact between materials (See **Fig.4**, hypothesis B, according to [20]); ($\tau_{zx\ average}$): average horizontal shear stress at the point of coordinates (x, y); (x): horizontal coordinate, of the point of analysis; (y): vertical coordinate, of the point of analysis; (a): half of the beam depth (See **Figs.3** and **9**); (b): half of the beam width (See **Figs.3** and **9**).

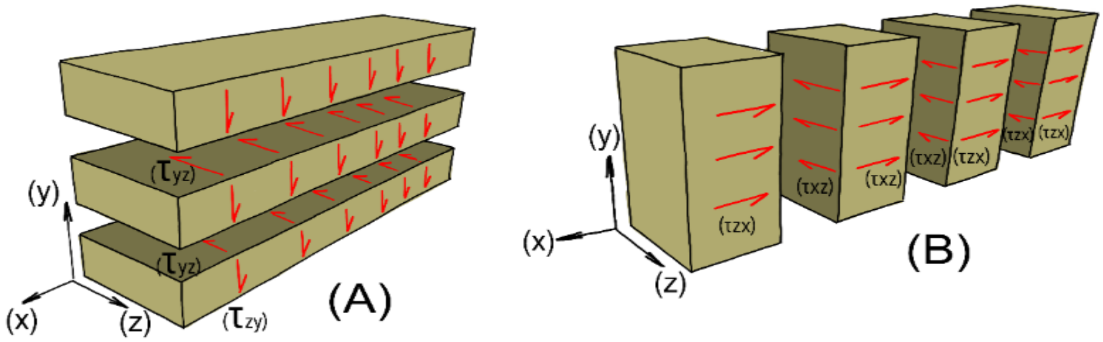

Fig.5. Case (A) corresponds to the typical case of composite beams of group B, in **Fig.1**, where the shear stresses develop between the horizontal planes of contact between the layers of materials, arranged one above the other. Case (B), moreover, corresponds to the case of shear stresses developed between the vertical planes of contact between the layers of materials, according to group (A).

3.2 Experimental validation of elastic theory

Table 1. Parameters of the materials used in the bending test, for the wood and steel members of the composite sections, according to [5].

Materials	values used in the experiment
Timber member (Domestic cedar, approximately 35 years old)	
Cross section:	38×140mm
Length:	3000mm
Density:	393 kg/m^3
Water Saturation (CNS14630 material standards code in Taiwan)	<15%
Mechanical Grade	E80
Cold-rolled stainless steel	
Elastic Modulus (CNS6183 material standards code in Taiwan)	(E)=203000N/mm^2
I-Shaped steel	144×70×2×2mm/length=3000mm
Steel plate	140×4mm/length=3000mm
Mechanical connectors	
Bolts	M14
Nails	CN-50/ length=76.2mm
Screws	#10-24/ length=88.90mm

Since the new calculation method proposed in this manuscript was developed entirely in the elastic domain, the four-point bending test method was used to validate its application. Experimental bending tests carried out by [4, 5] and [7-9-32-42] include specimens of timber beams, reinforced with steel plates and different mechanical connectors, from nails, bolts, to even epoxy-type adhesive. The bending experiment, performed by [5] on the behavior of the composite sections in **Fig. 6**, was carried out following the CNS 11301 standard in Taiwan. The results were obtained by examining the relationship between load (N) and deflection (mm), which were controlled by the 20 N/s load test. Six types of composite sections were used: Type A, B, C, D, and E, as detailed in Fig. 6 and **Table 1**.

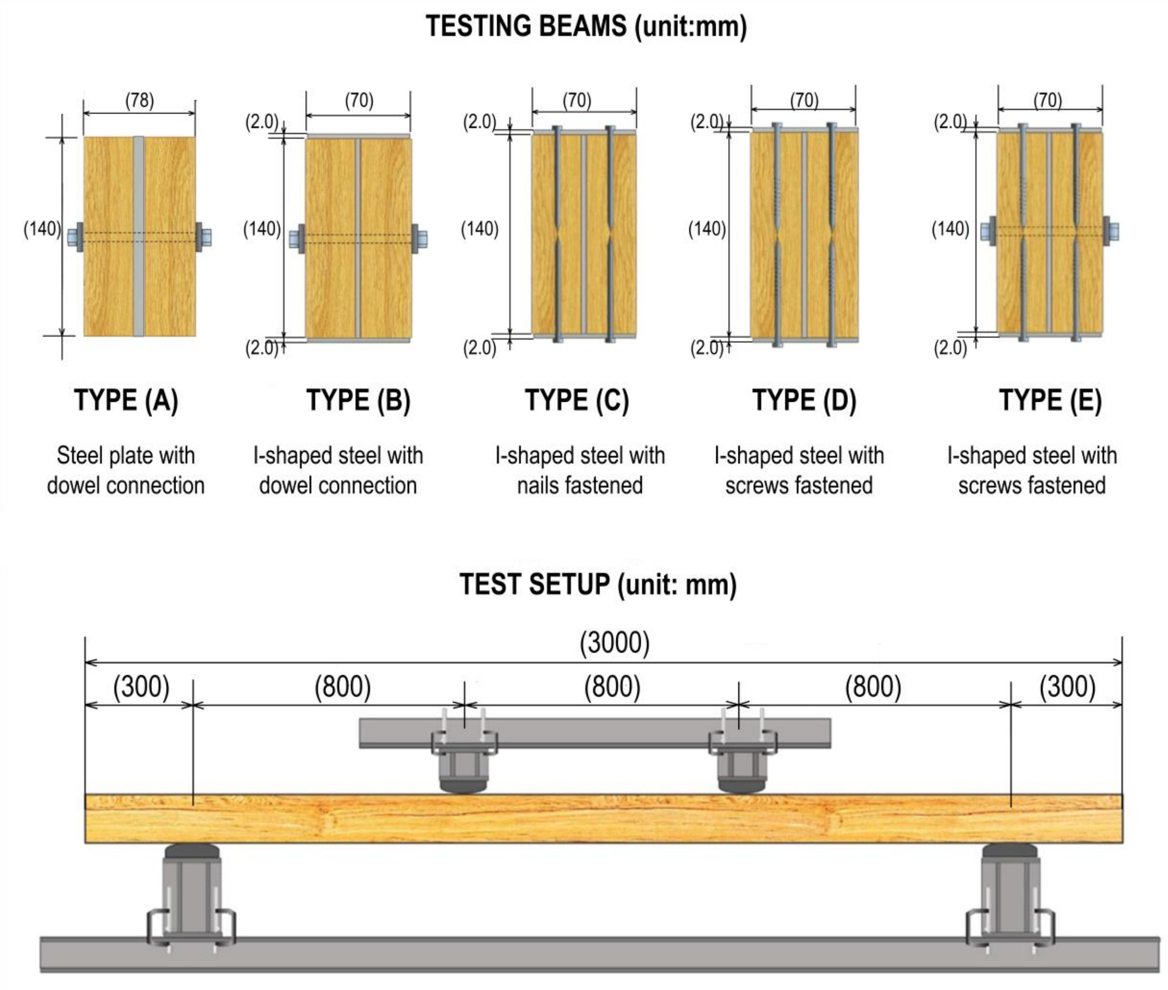

Fig.6. Flexural tests on timber beam specimens, reinforced with steel and various mechanical connections, such as nails, bolts, and combinations of both, conducted by [5]

3.3 Method of analysis based on the Theory of Elasticity

In composite beams such as flitch and built-up beams, to determine the spacing of horizontally arranged mechanical connectors, it is necessary to know the law of horizontal shear stresses (τ_{zx}). Based on the NDS 2005, the Rational Method dispenses with this parameter and uses other parameters instead. This has led to conservative results, as pointed out by several authors. The new Elastic Method is based on applying a simplified and novel equation derived from [20]. The new equation allows us to calculate the horizontal shear stresses (τ_{zx}) in composite beams of rectangular cross-section and allows us to describe the phenomenon of tension more faithfully. Naturally, this leads to a more generous spacing between the mechanical connectors. Moreover, the equation used is comparable in simplicity to the Zhuravsky-Collignon equation. Therefore, it allows us to solve, in a practical way, all cases of mixed beams of "Group A" in Fig. 1 with acceptable accuracy. It also calculates the horizontal shear stresses (τ_{zx})in "Group B" beams. Interestingly, in beams of considerable width, the values of horizontal stresses can equal or even exceed the vertical shear forces of the elementary shear theory (τ_{zy}). Currently, the NDS 2005 only includes the Zhuravsky-Collignon equation for calculating vertical stresses (τ_{zy}) within the Elastic Method, which the standard refers to as being based on Allowable Stress

Design (ASD). The new and simplified equation calculating horizontal shear stresses (τ_{zx}) is a practical and valuable addition to the standard.

Therefore, the problem will be analyzed through the simplified equations (4), (5), and (6) given by [20]. These equations allow for the simple calculation of the values reached by the horizontal shear stress (τ_{zx}) in the contact planes between the vertical layers of materials, including the maximum and average values. An important aspect to consider is the limitation of the model used by the Rational Method since it requires a simply supported beam with a uniform load. Therefore, to apply the Rational Method and solve other load states, an artifice is included to transform any load state into an equivalent load state with a uniform load. This transformation allows the Rational Method to be applied to any complex loading case. Moreover, thanks to the included transformation, it is possible to use the curves in **Table 9**. For this reason, the methods can be compared based on the design model of a simply supported beam with a uniform load. The advantages of the new Elastic Method over the Rational Method are:

- Horizontal shear stresses (τ_{zx}) can be determined simply and with acceptable accuracy;
- It applies to any loading condition;
- It applies to any support condition;
- It is applicable for static and hyperstatic beams;
- The distribution of the mechanical connectors can be varied and designed as desired.

In a complementary way, through the exact solution provided by the theory of elasticity, according to equations (2) and (3) given by [21, 22], respectively, the horizontal shear stress (τ_{zx}) developed in the vertical planes of contact between the layers of materials will be analyzed. Subsequently, the results obtained will be represented graphically as shown in **Figs. 14** and **15**, together with the results of the Rational Method.

3.4 Rational Method, in the two variants based on the NDS 2005 for Wood Construction.

The Rational Method to be developed in this work, in the variants of [16] and [19], contrary to what happens in the real stress phenomenon of the composite section, does not take into account the horizontal shear stresses (τ_{zx}) and uses other parameters such as the equivalent design force ($Z_{\perp}$) perpendicular to the grain (See **Fig. 11**), the reduced vertical distributed load and one of the two patterns suggested by the Empirical Method, to determine the spacing of the mechanical connectors. The Rational Method is developed in detail in tables 15,16, and 18, however, it is necessary to use the results of the necessary parameters for the procedure, which is done in **Tables 2, 3, 4, 5, 6,7, 9**, and **Figs 9, 10**, and **11**. In the Rational Method, when calculating the spacing between mechanical connectors, the staggered pattern is used for the composite section with a single metal plate. Therefore, when measuring the distance between bolts on the same line, it should be duplicated. Apparently, this is a practice that has been taken over from the Empirical Method only for the staggered pattern, since in the Rational Method, the spacing value determined corresponds to half of the value that is finally adopted. In the case where two metal plates are used, the spacing value calculated through the Rational Method coincides with the spacing value provided by the pattern of the Empirical Method.

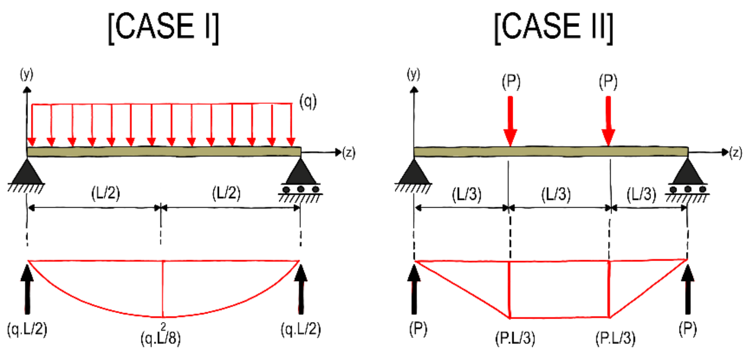

Fig. 7. Artifice for the determination of the equivalent load for load states other than uniformly distributed load. The maximum bending moment of case II is equated to the maximum bending moment produced by an equivalent uniform load. This allows the use of Fig. 10 for load states other than uniform load.

3.4.1 Calculation model used in the Rational Method.

The Rational Method requires the use of a specific design model to be applied: a simply supported beam, subjected to the action of a uniform load along its entire length, as can be seen in numerous design examples provided by [16], [18,19], [25] and [30] among many other authors. In practice, however, situations often arise in which the load states differ significantly from the case I (required by the Rational Method), shown in **Fig. 7**. An artifice presented by [23] is proposed to transform any load case, such as case II in **Fig. 7**, into load case I, through Eqs. (7) and (8), the maximum bending moments in cases I and II, respectively, are obtained. Therefore, by equating both equations, it is possible to determine an equivalent uniform load given by Eq. (9), which generates a bending moment equivalent in magnitude to that produced in Case II but for a beam as in Case I. Through this artifice, it is possible to extend the method to any load state for a simply supported beam.

For Case I (required for the Rational Method):

$$M_{\underset{1}{\max}} = \frac{(q)(L^2)}{8} \tag{7}$$

For Case II:

$$M_{\underset{\text{II}}{\max}} = (P)\left(\frac{L}{3}\right) \tag{8}$$

To obtain an equivalent uniform load for Case II: $M_{\underset{\text{I}}{\max}} = M_{\underset{\text{II}}{\max}}$

$$q_{\text{equivalent}} = \left[(P)\left(\frac{L}{3}\right)\right]\left[\frac{8}{(L^2)}\right] \tag{9}$$

3.5 Empirical Method.

As mentioned above, to solve the problem of composite beams, such as flitch beams, where the layers of materials are linked through horizontally arranged mechanical connectors, it is necessary to know the law of horizontal shear stresses (τ_{zx}). The Empirical Method only consists of a series of practical recommendations for the distribution and spacing pattern of horizontally arranged mechanical connectors, which are used in composite sections such as flitch beams, depending on whether they are single or double plates, according to [16]. On the other hand, [15] also points out a rule of thumb, based on his experience, for determining the spacing of mechanical connectors: forty times the plate spacing for bolts on the compressed side of the beam, while eighty times for the tension side. According to [15], the diameter of the bolts is also determined by a rule of thumb, the result of experience, without a theoretical basis. 1/2 in. bolts for plates less than 1/2 in. thick, 5/8 in. bolts for plates up to 3/4 in. thick, and 3/4 in. bolts for plates up to 1 in. thick.

The recommendations of the Empirical Method lack any theoretical support, as they come from experience gained over the years as a result of trial and error. It is not known precisely at what point in history the composite beams, in particular the so-called flitch and built-up beams, were first used, but it is known that the first calculation based on the Empirical Method was published for the first time in 1883, according to [11]. When elastic calculation and strength of materials were still under development, namely in the early to mid-19th century, many of the tools available today were lacking, so even the understanding of the phenomenon of vertical shear stresses (τ_{zy}) in beams was not fully understood.

In 1844, Russian engineer Dmitry Ivanovich Zhuravsky developed an approximate method for calculating vertical shear stresses (τ_{zy}) in timber beams and beams reinforced with rivets, which was formally published in 1854. He also applied his method of analysis to built-up iron beams and showed that the distance between rivets could be calculated if the shearing force per rivet was known. He also showed that the number of rivets used in bridges with tubular sections could be reduced considerably, since he demonstrated that the shearing force decreases from the supports towards the center of the span, so the distance of rivets can be increased without impairing the strength of the tubular section, according to [33,34] and [38]. The famous French mathematician and engineer Barré de Saint-Venant praised Zhuravksy's solution and would later develop an exact solution to the problem, circa 1856. However, the rigorousness of the solution covered a small number of cross-sections of simple shapes and made it difficult to apply it to more complex cases, so engineers were forced to use the approximate theory presented by Zhuravsky. Finally, in 1869, the French engineer Édouard Collignon would publish a more general version of Zhuravsky's method, resulting in the famous equation that is known as the Collignon-Zhuravsky equation (See Eq.1) [29]. Certainly, the Empirical Method provided valuable guidelines to designers and builders to assemble and use this type of composite beam, with excellent structural behavior and a generous safety margin. Although Saint-Venant had already published a solution to the problem in his 1856 treatise on elasticity, as Timoshenko mentions, although the procedure was correct, some calculation errors were detected [41]. This led Professor Reissner, in 1946, to publish the exact solution of the problem with corrections [21].

However, possibly due to factors attributed to its complexity and scarce diffusion, it would go unnoticed, to such an extent that even today, the problem continues to be approached in a way that does not differ substantially from those years. As can be seen in specialized publications such as [16] and [30], no approach takes into consideration the law of horizontal shear stresses, as proposed by the new Elastic Method. In **Fig. 8**, the guidelines for the application of the Empirical Method according to [16] are presented. Reference is only made to two cases of flitch-beam, with single and double metal plates, with the corresponding distribution pattern and spacing.

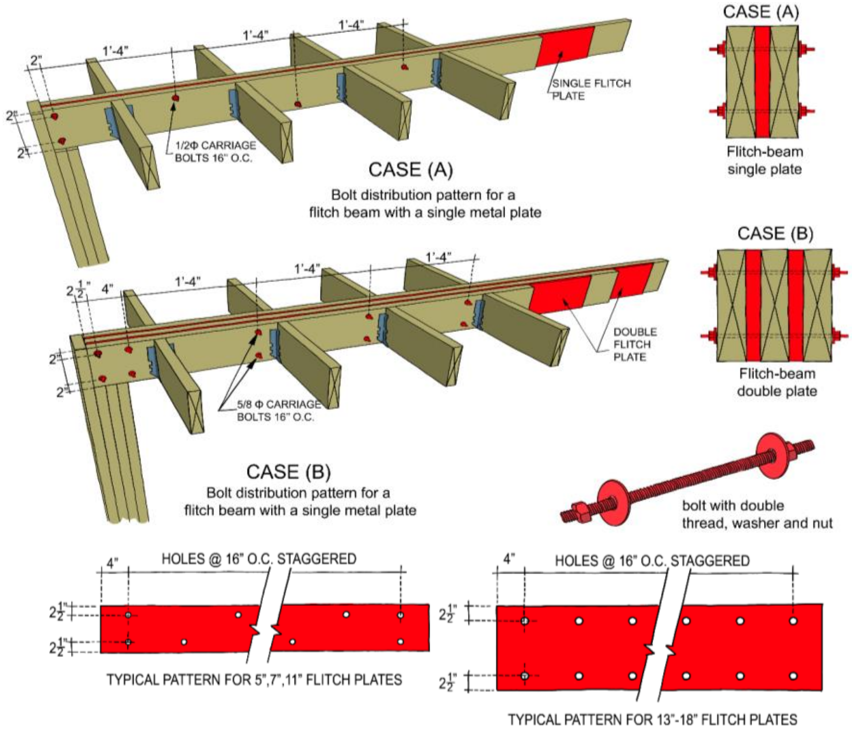

Fig. 8. According to [15], the Empirical Method uses spacings validated by satisfactory structural performance over the years. However, no theoretical justification is available, to the extent that the Rational Method must use one of the shown patterns to determine the spacing value, so it is not a pure method on a theoretical level. Steel plates with defined distribution and spacing patterns are available in the US and Canadian markets.

4 Calculation Procedure.

The strength parameters of the materials used in the composite beam are given in **Table 1**. Since in most practical application situations the simply supported beam with a uniformly distributed load is used, this structural model has been chosen to develop the analysis. Therefore, for a simply supported beam, the expressions corresponding to the allowable load due to the maximum bending moment and the maximum deflection at the center of the beam are given by the expressions in **Table 2**.

In (**Figs.9** and **10**), six sections of flitch beams (using three types of steel plate thicknesses combined to obtain a total of 6 sections) were analyzed. In addition, nine scenarios with different spans and loads were considered for each section. Thus, 54 analysis cases were obtained and summarised in the curves (See **Fig. 10**). Each curve provides the allowable uniform load for a simply supported beam with a given free span and/or vice versa, depending on the situation or the requirements of the structural designer. This makes the calculation process incredibly fast. It is important to mention that, due to the close spacing of the acting loads (due to the Wood-frame construction system, see **Fig. 8**), it is usual and has proven to give good results to consider a uniform load distributed over the length of the beam instead of point loads. This structural design model is widely used in the field of professional engineering practice in Canada, the USA, and the UK, among other countries, to apply the Rational Method. Numerous authors, such as [16], [18, 19], and [30] among many others, agree that the design model with uniform loading over the entire length of the beam is appropriate. Before the application of the Rational Method and the Elastic Methods, it is necessary to calculate the resistant modulus of the cross-section. For this purpose, it is necessary to homogenize the composite cross-section, using the parameter "*n*" of **Table 2**, which allows transforming the metal plate into a wooden plank of equivalent thickness, as shown in **Fig. 9**. Then, from the homogenized cross-section, it is possible to calculate the resistant modulus of the complete cross-section, according to **Table 4**.

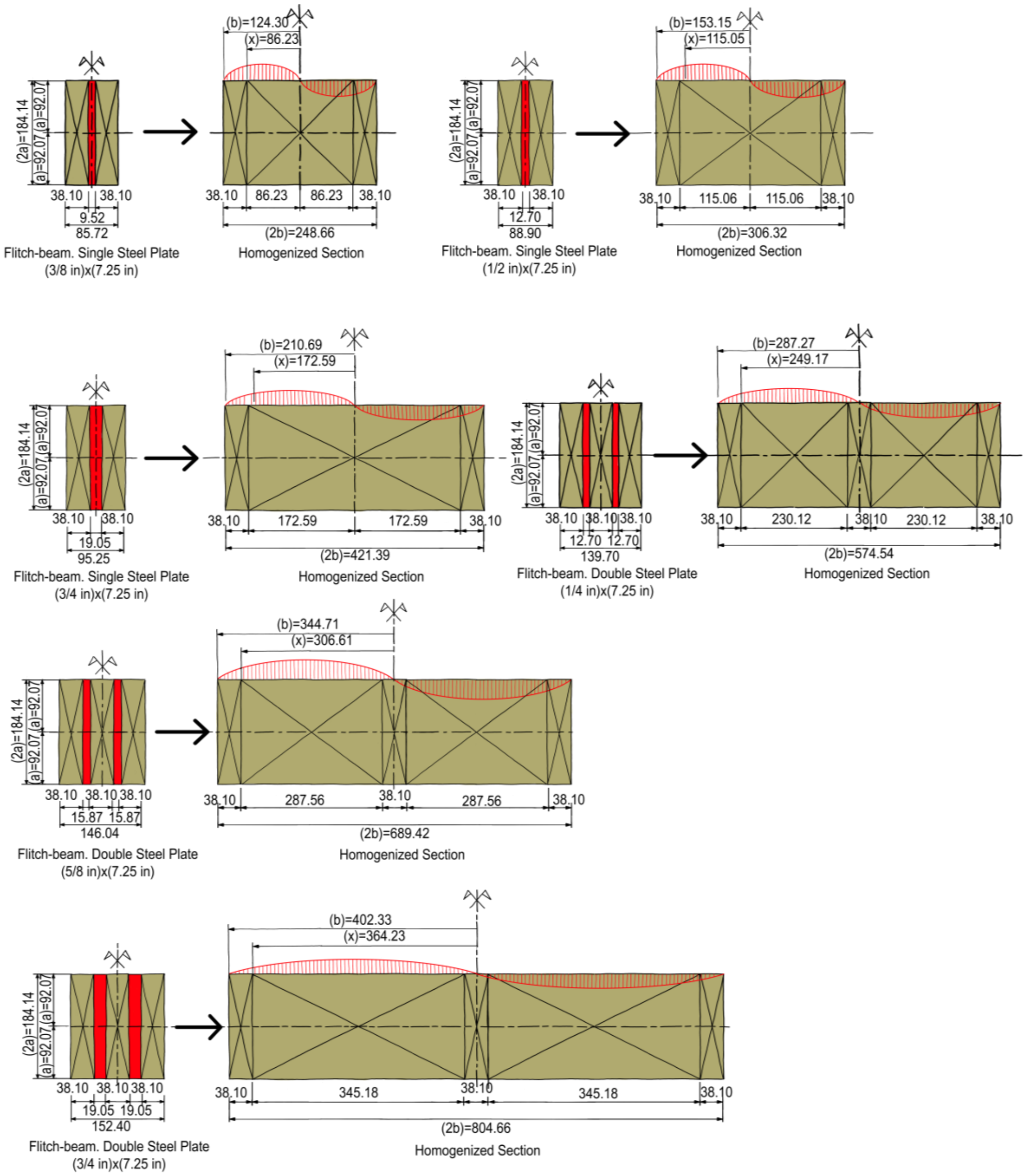

Fig. 9. Homogenization of cross-sections for flitch beams, with varying number and thickness of metal plates. Additionally, the calculation of the joint distance "*x*" is presented for the calculation of the horizontal shear stress, to verify the separation of the mechanical connectors (bolts), through the solution proposed by the new method and the exact and complex solution given by the theory of elasticity.

Table 2. Strength Parameters of the Materials Used in the Composite Beam.

Material Properties and Factors for the Design	
Steel Plate (ASTM A36)Modulus of Elasticity:	$(E_s) = 29000000$ psi $\approx 203{,}87$ kN/mm^2
Wooden Planks- Modulus of Elasticity: (Douglas Fir Larch#2) Table 4A from [17]	$(E_w) = 1600000$ psi $\approx 11{,}25$ kN/mm^2
Wooden Planks-Design Value for Bending (Douglas Fir Larch#2) Table 4A from [17]	$(F_b) = 900$ $psi \approx 6{,}327x10^{-3}$ kN/mm^2
Coefficient Homogenization:	$(n) = E_s/E_w \approx 18.13$
Specific Gravity: Table 11.3.2A from [17]	$(G) = 0.50$
Adjustment factors Beam Stability Factor Size Factor Moisture or Wet Factor Load Duration Factor Temperature Factor Repetitive Member Factor Bearing Stress Factor Flat Use Factor Incision factor	$(C_L) = 1.00$ $(C_F) = 0.90$ $(C_M) = 1.00$ $(C_D) = 1.00$ $(C_t) = 1.00$ $(C_r) = 1.15$ $(C_b) = 1.00$ $(C_{fu}) = 1.00$ $(C_i) = 1.00$
Allowable Bending Stress	$F_b' = F_b(C_L)(C_F)(C_M)(C_D)(C_t)(C_r)(C_b)(C_{fu})(C_i) \approx F_b$
Dowel Bearing Strengths (Table 11.3.2 from [17]) Parallel to the grain Perpendicular to the grain	 $(F_{e\parallel})$=3150 psi$\approx 2.214x10^{-2}$kN/mm^2 $(F_{e\perp})$=5600 psi$\approx 3.937x10^{-2}$ kN/mm^2
Poisson's ratio According to [21],[36] and [39]	$(\nu)\approx 0.30$

For the calculation of the reference design values, necessary for both the rational and the Elastic Methods, the recommendations of the American standard NDS 2005 (Art. 4.1.4 'Moisture service condition of lumber') have been strictly followed. The standard establishes the environmental conditions for most of the covered structures, where the moisture content is assumed to be $\leqslant$ 19%. To take into consideration long periods of exposure to moisture conditions, the design values for lumber in dry service conditions were multiplied by the temperature (C_t) and service moisture (C_M) factors for temperature values T $\leqslant$ 100 °F. (See **Table 2**).

Table 3. Strength and Deformation Conditions, to determine the Allowable Distributed Load

Conditions	Determination of allowable load
Due to the allowable bending: $q_1 = (\frac{8W_{w+s}F_b'}{L})^{1/2}$	$if: \; q_1 < q_2 \rightarrow q_{\text{allowable}} = q_1$
Due to the allowable displacement $q_2 = (\frac{L}{360})(\frac{384}{5})(\frac{EI}{L^4})$	$if: \; q_2 < q_1 \rightarrow q_{\text{allowable}} = q_2$

Note: (q_1): allowable uniform load, due to bending, for a simply supported beam (See **Fig. 7** Case I); (W_{w+s}): modulus of resistance of the cross section of the homogenized composite material (See Table 4); (L): span between supports, for a simply supported beam (See **Table 5** and **Fig. 10**); (F_b'):Allowable Bending Stress (See **Table 2**); (q_2): allowable load, uniformly distributed, due to the allowable vertical displacement, for a simply supported beam (See **Fig. 7** Case I); (I): barycentric moment of inertia of the composite cross-section, homogenized (See **Table 4**); (E): modulus of elasticity of the homogenized cross section (See **Table 2** $E = E_w$).

Table 4. Geometric Properties of the Homogenized Rectangular Cross-Sections

Steel Plate Thickness	Steel Plate depth ($h_s = 184.15$mm)		Wooden Planks ($t_w = 38.10$mm) ($h_w = 184.15$mm)		Composite Section		Ratio coefficient
t_s [mm]	Equivalent thickness $t_s^* = (t_s)(n)$ [mm]	$W_s = \frac{t_s^* h_s^2}{6}$ [mm³]	$W_w = \frac{t_w h_w^2}{6}$ [mm³]	$W_s + W_w$ [mm³]	$I_s + I_w$ [mm^4]	$(k) = \frac{W_s}{W_s + W_w}$	width/depth ratio $\left(\frac{b}{a}\right)$
9.52	172.64	9.76E+05	1.82E+04	6.32E+05	9.37E+09	0.69	1.35
12.70	230.19	1.30E+06	1.82E+04	1.18E+06	2.15E+10	0.75	1.66
19.05	345.28	1.95E+06	1.82E+04	3.07E+06	7.71E+10	0.82	2.29
2x12.70	460.38	2.60E+06	6.13E+04	7.79E+06	1.93E+12	0.80	3.12
2x15.87	575.47	3.25E+06	6.13E+04	1.35E+07	4.28E+12	0.83	3.75
2x19.05	690.56	3.90E+06	6.13E+04	2.14E+07	8.30E+12	0.86	4.37

Note: (h_s): depth of the steel plate (See **Fig.9**); (t_s): thickness of the steel plate (See **Fig.9**); (h_w): depth of the wooden plank (See **Fig.9**); (t_w): thickness of the wooden plank (See **Fig.9**); (t_s^*): homogenized steel plate thickness (See **Fig.9**); (n): coefficient of homogenization (See **Table 2**); (W_s): modulus of resistance of steel cross-section (steel plate); (W_w): modulus of resistance of wood cross-section (wooden planks); (I_s): barycentric moment of inertia of a steel cross-section (steel plate); (I_w): barycentric moment of inertia of a wood cross-section (wooden planks); (k): coefficient of ratio between the resistant modulus of the steel cross-section and the total cross-section; (W_{w+s}): modulus of resistance of the cross section of the homogenized composite material; (b/a): coefficient of ratio width/depth of the complete, homogenized cross-section; (a): half of the beam depth (See **Figs.3** and **9**); (b): half of the beam width (See **Figs.3** and **9**)

Table 5. Allowable Uniform Load “(q)”, for different combinations and thicknesses of metal plates combined with wood planks. The load “q” is used in the Rational Method in the variants of [16] and [19].

span “L” [m]	Allowable Uniform Load "(q)" [kN/m]					
	Composite Beam with single steel plate of depth $h_s = 184.15$ mm and variable thickness			Composite Beam with double steel plate of depth $t_s = 184.15$mm and variable thickness		
	(9.52) [mm]	(12.70) [mm]	(19.05) [mm]	(2×12.70) [mm]	(2×15.87) [mm]	(2×19.05) [mm]
1.78	22.52	27.73	38.15	52.01	62.43	72.84
2.03	17.24	21.23	29.20	39.82	47.79	55.77
2.29	13.62	16.77	23.08	31.46	37.76	44.06
2.54	11.04	13.59	18.69	25.48	30.59	35.69
2.79	9.12	11.23	15.45	21.06	25.28	29.50
3.05	7.66	9.44	12.98	17.70	21.24	24.79
3.30	6.53	8.04	11.06	15.08	18.10	21.12
3.56	5.63	6.93	9.54	13.00	15.61	18.21
3.81	4.90	6.04	8.31	11.33	13.59	15.86

Note: (q): allowable uniform load for composite sections. It includes the single steel plate case and the double steel plate case. Both cases with different plate thicknesses; (L): span between supports, for a simply supported beam (See **Fig. 7** Case I); (h_s): depth of the steel plate (See **Fig. 9**); (t_s): thickness of the steel plate (See **Fig. 9**).

In the Rational Method, described by [16], since the horizontal shear stresses are not used, the vertical load ‘q’ that originates the shear effect on the bolt shank is assumed. Since the load ‘q’ is perpendicular to the grain, the equation for the single or double shear case described in [17-24] must be adopted. In the method of [16], based on [17], the double shear mechanism is considered. According to [16], although the method is very practical, it leads to extremely conservative results. The lack of a solid theoretical framework is mentioned by [15], and this has inevitably led to different results, which vary considerably from one designer to another, depending on their criteria. This can easily be seen in the method of [19], which is also a variant of the Rational Method based on [17]. On the other hand, in the theory of elasticity, the homogenized cross-section is assumed to behave as a deformable solid under the action of loads. Therefore, this leads to accepting the existence of horizontal shear forces, which are generated as a result of slippage between the vertically arranged planks in contact with each other. For this reason, it is necessary to use the equation corresponding to the design parameter $Z_{\parallel}$, when the stress is parallel to the wood grain (See **Fig.12**).

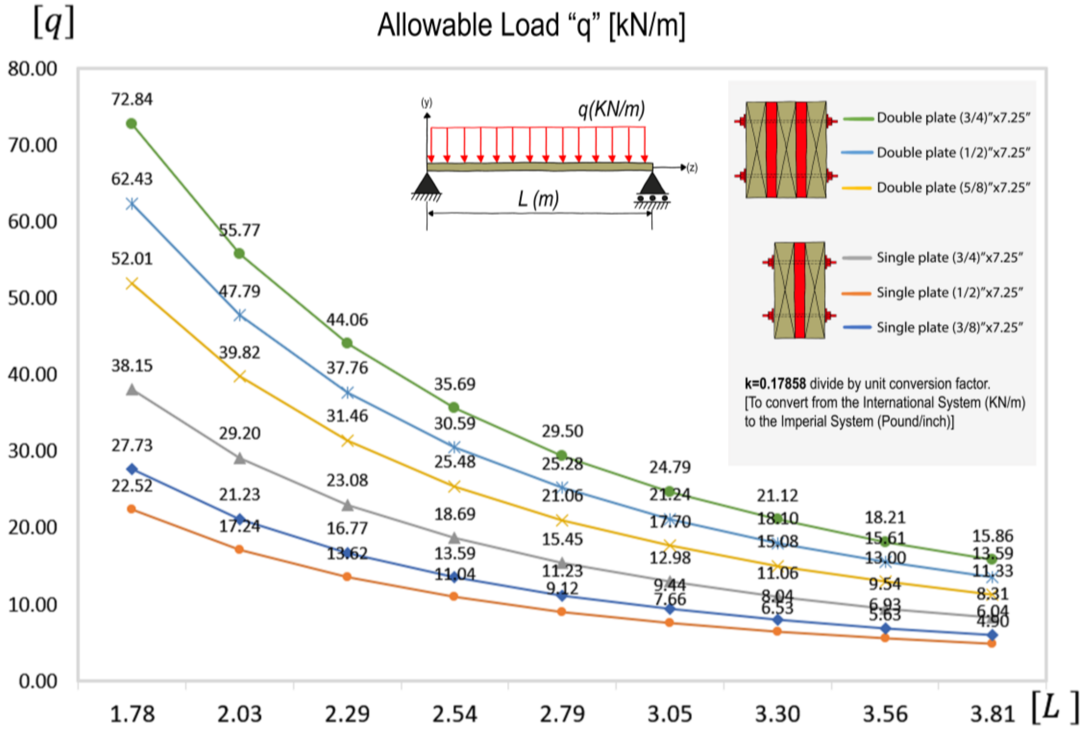

Fig. 10. The allowable load curves, for different spans, of a simply supported beam under the action of a uniformly distributed load correspond to the values calculated in Table 4.

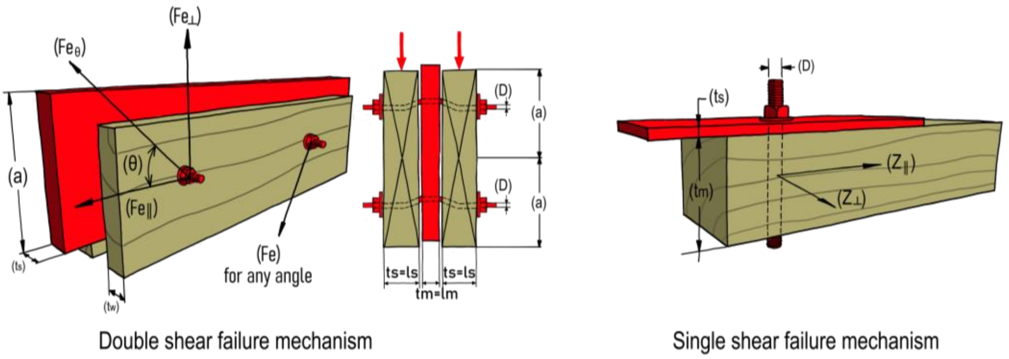

Fig. 11. Variants of the Rational Method based on [17]. The double shear failure mechanism, considered by [16]. While a single shear failure mechanism is considered by [19] according to [17].

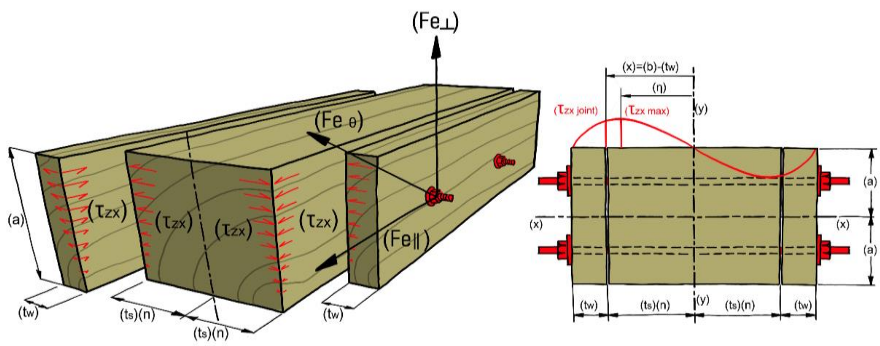

Fig. 12. Failure mechanism, double shear for the case of a single plate, considered according to the Theory of Elasticity from [20], [21], and [22]. For the two-plate case, it would correspond to the multiple-shear case, but for this research work, it will be considered only as the double-shear case.

Table 6. Reduction of the uniform load as a function of the ratio coefficient (k), for application of the Rational Method in the variant of [16].

Reduced Allowable Uniform Load $q_{\text{reduced}} = (q)(k)$ [kN/m].						
Span "L" [m]	Composite Beam with single steel plate of depth $h_s = 184.15\text{mm}$ and variable thickness t_s (mm)			Composite Beam with double steel plate of depth $h_s = 184.15\text{mm}$ and variable thickness t_s (mm)		
	$t_s = 9.52$ k=0.69	$t_s = 12.70$ k=0.75	$t_s = 19.05$ k=0.82	$2t_s = 2x12.70$ k=0.80	$2t_s = 2x15.87$ k=0.83	$2t_s = 2x19.05$ k=0.86
1.78	10.78	15.62	25.62	33.33	43.23	53.75
2.03	8.25	11.96	19.62	25.52	33.10	41.15
2.29	6.52	9.45	15.50	20.16	26.15	32.51
2.54	5.28	7.66	12.56	16.33	21.18	26.34
2.79	4.37	6.33	10.38	13.50	17.51	21.77
3.05	3.67	5.32	8.72	11.34	14.71	18.29
3.30	3.13	4.53	7.43	9.66	12.53	15.58
3.56	2.70	3.91	6.41	8.33	10.81	13.44
3.81	2.35	3.40	5.58	7.26	9.41	11.71

Note: ($q_{reduced}$): uniform allowable load, reduced by the coefficient "k", for the application of the Rational Method; (q): allowable uniform load, for composite cross-sections, with single and double plate, considering different thicknesses in each case (See **Table 5** and **Fig. 10**); (L): span between supports, for a simply supported beam (See **Fig. 7** Case I); (h_s): depth of the steel plate (See **Fig. 9** and **Table 5**); (t_s): thickness of the steel plate (See **Fig. 9** and **Table 5**); (k): coefficient of ratio between the resistant modulus of the steel cross-section and the total cross-section (This coefficient has been calculated for each case in **Table 4**.)

Table 7. According to [17], the Yield Limit Equations are given in the following Table. The reference design value of the mechanical connector ($Z_{\perp}$) corresponds to the smallest value in the equations. This value will be used in the Rational Method, in the variant of [16].

Connection Yield Modes (Double Shear)	Yield Limit Equations According to [NDS, 2005]	Reference Design Value $Z_{\perp}$ **[kN]** ($Fe_{\perp}$=3150 psi) for Composite Beams for different thicknesses of metal plates (bolt diameter=1/2 in.)		
		3/8 (in)	1/2 (in)	5/8 (in)
Mode I_m $(R_d = 4K_\theta)$:	$Z = \frac{(D)(l_m)(F_{em})}{R_d}$	14.80	19.73	24.66
Mode $\text{I}_s(R_d = 4K_\theta)$:	$Z = \frac{(2D)(l_s)(F_{es})}{R_d}$	4.29	4.29	4.29
Mode III_s $(R_d = 3.2K_\theta)$:	$Z = \frac{(2K_3)(D)(l_s)(F_{em})}{(2+R_e)R_d}$	**3.95**	**3.95**	**3.95**
Mode IV_s $(R_d = 3.2K_\theta)$:	$Z = \frac{2(D^2)}{R_d}\sqrt{\frac{2(F_{em})(F_{yb})}{3(1+R_e)}}$	5.41	5.41	5.41

Note: (R_d): reduction term. (See **Table 9**); (θ): maximum angle of load to grain $(0 \leq \theta \leq 90)$. (See **Table 9**); (K_θ): coefficient to calculate the reduction factor, as a function of the angle of the load to the grain. (See **Table 9**); (Z): reference design value, to calculate $Z_{\parallel}$ or $Z_{\perp}$, depending on the case of analysis; ($Z_{\perp}$): reference design value, perpendicular to the grain (See **Fig.11**); ($Fe_{\perp}$): dowel bearing strength, perpendicular to the grain (See **Table 9**); (D): bolt diameter (See **Table 9**); (l_m): $m$ain member dowel bearing length in inches (See **Fig.11**); (l_s): $s$ide member dowel bearing length in inches (See **Fig.11**); (F_{em}): main member dowel bearing strength in psi (See **Table 9**); (F_{es}): side member dowel bearing strength in psi (See **Table 9**); (R_e): ratio between main member dowel bearing strength and the side member dowel bearing strength (See **Table 9**); (R_t): ratio between main member dowel bearing length and side member dowel bearing length (See **Table 9**); (F_{yb}): dowel bending yield strength (See **Table 9**); (k_3): coefficient for the calculation of double shear, for yield mode IV (See **Table 9**).

Table 8. According to [17], the Yield Limit Equations are given in the following Table. The reference design value of the mechanical connector ($Z_{\parallel}$) corresponds to the smallest value of the equations to be used in the Elastic Methods, [20], [21], and [22].

Connection Yield Modes (Double Shear)	Yield Limit Equations According to [NDS, 2005]	Reference Design Value $Z_{\parallel}$ [kN] ($Fe_{\parallel} = 5600\ psi$) for Composite Beams for different thicknesses of metal plates (bolt diameter=1/2 in.) 3/8 (in)	1/2 (in)	5/8 (in)
Mode I_m $(R_d = 4K_\theta)$:	$Z = \frac{(D)(l_m)(F_{em})}{R_d}$	18.50	24.66	30.83
Mode $\mathrm{I}_s(R_d = 4K_\theta)$:	$Z = \frac{(2D)(l_s)(F_{es})}{R_d}$	9.53	9.53	9.53
Mode III_s $(R_d = 3.2K_\theta)$:	$Z = \frac{(2K_3)(D)(l_s)(F_{em})}{(2+R_e)R_d}$	7.10	7.10	7.10
Mode IV $(R_d = 3.2K_\theta)$:	$Z = \frac{2(D^2)}{R_d}\sqrt{\frac{2(F_{em})(F_{yb})}{3(1+R_e)}}$	8.90	8.90	8.90

Note: (R_d): reduction term. (See **Table 9**); (θ): maximum angle of load to grain $(0 \le \theta \le 90)$.(See **Table 9**); (K_θ): coefficient to calculate the reduction factor, as a function of the angle of the load to the grain. (See **Table 9**); (Z): reference design value, to calculate $Z_{\parallel}$ or $Z_{\perp}$, depending on the case of analysis; ($Z_{\parallel}$): reference design value, parallel to the grain (See **Fig. 12**); ($Fe_{\parallel}$): dowel bearing strength, parallel to the grain (See Table 9); (D): bolt diameter (See **Table 9**); (l_m): main member dowel bearing length in inches (See **Fig 12**); (l_s): side member dowel bearing length in inches. (See **Fig12**); (F_{em}): main member dowel bearing strength in psi (See **Table 9**); (F_{es}): side member dowel bearing strength in psi (See **Table 9**); (R_e): ratio between main member dowel bearing strength and the side member dowel bearing strength (See **Table 9**); (R_t): ratio between main member dowel bearing length and side member dowel bearing length (See **Table 9**); (F_{yb}): dowel bending yield strength (See **Table 9**); (k_3): coefficient for the calculation of double shear, for yield mode IV. (See **Table 9**).

Table 9. Auxiliary calculation parameters, for the calculation of the minimum design value of the bolt, are required for [16], [19], [20], [21], and [22].

Bolt diameter	$(D) = \frac{1}{2}$ in.	(See Appendix L, Table L1 from [17])
Dowel Bending Yield Strength	(F_{yb})=45000 psi	(See Table I1 from [17]
Reduction Term	$(R_d)=4K_\theta$	for Modes (I_m) and (I_s)
Reduction Term	$(R_d)=3.2K_\theta$	for Modes (III_m) and (IV)
Maximum angle of load to grain	$(0 \le \theta \le 90)$	
Factor	$(K_\theta)=1+0.25\left(\frac{\theta}{90}\right)$	(See Fig.11 and 12)
Main member dowel bearing length	(l_m), in.	(See Fig.11 and 12)
Side member dowel bearing length	(l_s), in.	(See Fig.11 and 12)
Main member dowel bearing strength	(F_{em}), psi.	(See Table 11B from [17]; 87000 psi for ASTM A 36)
Side member dowel bearing strength	$F_{es} = F_{e\parallel}$ for $Z_{\parallel}$ $F_{es} = F_{e\parallel}$ for $Z_{\perp}$	(Table 11.3.2, for a bolt diameter of ½ in., Douglas Fir-Larch Specie, and specific gravity G=0,50)
Ratio between main member dowel bearing strength and the side member dowel bearing strength	$R_e=\frac{F_{em}}{F_{es}}$	
Ratio between main member dowel bearing length and side member dowel bearing length	$R_t = \frac{l_m}{l_s}$	(See Fig.11 and 12)
Coefficient for the calculation of double shear, for yield mode IV	$K_3 = -1+\sqrt{\frac{2(1+R_e)}{R_e}+\frac{2F_{yb}(1+2R_e)D^2}{3F_{em}l_m^2}}$	
Dowel bearing strength	$Fe_{\parallel} = 5600$ psi	See Table 11.3.2 from [17]
Dowel bearing strength	$Fe_{\perp} = 3150$ psi	See Table 11.3.2 from [17]

Table 10. According to the $\tau_{zx\ joint}$ stresses developed at the joint, by [21].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q=\frac{(q)(L^*)}{2}$ [kN] (L^*=3.81m)	$(\tau_{zx\ joint})$ [kN/m²]	$\left(\frac{\tau_{zx\ joint}}{2}\right)$ [kN/m²]	$T_{zx}=\left(\frac{\tau_{zx\ joint}}{2}\right)(a)$ [kN/m]	$Z \parallel$ [kN]	Spacing $e=\left(\frac{Z \parallel}{T_{zx}}\right)$ [m]
Single	1.35	4.90	9.34	42.92	21.46	1.98	7.10	3.59
Steel	1.66	6.04	11.50	71.42	35.71	3.29	7.10	2.16
Plate	2.29	8.31	15.83	102.76	51.38	4.73	7.10	1.50
Double	3.12	11.33	21.58	157.94	78.97	7.27	7.10	0.98
Steel	3.75	13.59	25.90	199.21	99.60	9.17	7.10	0.77
Plate	4.37	15.86	30.22	241.07	120.53	11.10	7.10	0.64

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load (See **Table 5** and **Fig. 10**); (L^*): span between supports (See **Table 5** and **Fig. 10**); (Q): vertical shear at the support, for a simply supported beam (See **Fig. 7** Casse I); ($\tau_{zx\ joint}$): horizontal shear stresses, at a point of coordinates (x, y), belonging to the vertical plane of contact between materials (See Eq. (2)); (T_{zx}): average horizontal shear force per linear meter; ($Z_{\parallel}$): reference design value, parallel to the grain. (See **Table 8**); (e): spacing between mechanical connectors, horizontally arranged.

Table 11. According to the $\tau_{zx\ joint}$ stresses developed at the joint, by [22].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q=\frac{(q)(L^*)}{2}$ [kN] (L^*=3.81m)	$(\tau_{zx\ joint})$ [kN/m²]	$\left(\frac{\tau_{zx\ joint}}{2}\right)$ [kN/m²]	$T_{zx}=\left(\frac{\tau_{zx\ joint}}{2}\right)(a)$ [kN/m]	$Z \parallel$ [kN]	Spacing $e=\left(\frac{Z \parallel}{T_{zx}}\right)$ [m]
Single	1.35	4.90	9.34	52.35	26.17	2.41	7.10	2.95
Steel	1.66	6.04	11.50	75.28	37.64	3.47	7.10	2.05
Plate	2.29	8.31	15.83	121.60	60.80	5.60	7.10	1.27
Double	3.12	11.33	21.58	192.72	96.36	8.87	7.10	0.80
Steel	3.75	13.59	25.90	247.32	123.66	11.39	7.10	0.62
Plate	4.37	15.86	30.22	300.47	150.24	13.83	7.10	0.51

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load (See **Table 5** and **Fig. 10**); (L^*): free span between supports (See **Table 5** and **Fig. 10**); (Q): vertical shear at the support, for a simply supported beam (See **Fig. 7** Case I); ($\tau_{zx\ joint}$): horizontal shear stress, at a point of coordinates (x, y), belonging to the vertical plane of contact between materials (See Eq. (3)); (T_{zx}): average horizontal shear force per linear meter; ($Z_{\parallel}$): reference design value, parallel to the grain. (See **Table 8**); (e): spacing between mechanical connectors, horizontally arranged.

Table 12. According to the $\tau_{zx\ joint}$ stresses developed at the joint, by [20].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q=\frac{(q)(L^*)}{2}$ [kN] (L^*=3.81m)	$(\tau_{zx\ joint})$ [kN/m²]	$\left(\frac{\tau_{zx\ joint}}{2}\right)$ [kN/m²]	$T_{zx}=\left(\frac{\tau_{zx\ joint}}{2}\right)(a)$ [kN/m]	$Z \parallel$ [kN]	Spacing $e=\left(\frac{Z \parallel}{T_{zx}}\right)$ [m]
Single	1.35	4.90	9.34	98.82	49.41	4.55	7.10	1.56
Steel	1.66	6.04	11.50	107.00	53.50	4.93	7.10	1.44
Plate	2.29	8.31	15.83	116.64	58.32	5.37	7.10	1.32
Double	3.12	11.33	21.58	123.50	61.75	5.69	7.10	1.25
Steel	3.75	13.59	25.90	126.70	63.35	5.83	7.10	1.22
Plate	4.37	15.86	30.22	128.94	64.47	5.94	7.10	1.20

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load (See **Table 5** and **Fig. 10**); (L^*): span between supports (See **Table 5** and **Fig. 10**); (Q): vertical shear at the support, for a simply supported beam (See **Fig. 7** Case I); ($\tau_{zx\ joint}$): horizontal shear stresses, at a point of coordinates (x, y), belonging to the vertical plane of contact between materials (See Eq. (6)); (T_{zx}): average horizontal shear force per linear meter; ($Z_{\|}$): reference design value, parallel to the grain. (See **Table 8**); (e): spacing between mechanical connectors, horizontally arranged.

Table 13. According to the $\tau_{\text{zx Average}}$ stresses, by [20].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q=\frac{(q)(L^*)}{2}$ [kN] (L^*=3.81m)	$(\tau_{\text{zx joint}})$ [kN/m^2]	$\left(\frac{\tau_{\text{zx joint}}}{2}\right)$ [kN/m^2]	$T_{zx}=\left(\frac{\tau_{\text{zx joint}}}{2}\right)(a)$ [kN/m]	$Z \|$ [kN]	Spacing $e=\left(\frac{Z \|}{T_{zx}}\right)$ [m]
Single	1.35	4.90	9.34	77.49	38.74	3.57	7.10	2.00
Steel	1.66	6.04	11.50	95.41	47.70	4.39	7.10	1.62
Plate	2.29	8.31	15.83	131.25	65.62	6.04	7.10	1.18
Double	3.12	11.33	21.58	178.95	89.48	8.24	7.10	0.86
Steel	3.75	13.59	25.90	214.79	107.40	9.89	7.10	0.72
Plate	4.37	15.86	30.22	250.63	125.32	11.54	7.10	0.62

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load (See **Table 5** and **Fig. 10**); (L^*): span between supports (See **Table 5** and **Fig. 10**); (Q): vertical shear at the support, for a simply supported beam (See **Fig.7** case I); ($\tau_{\text{zx average}}$): average horizontal shear stresses, at a point of coordinates ($x=b/2$; $y=a$). (See Eq. (4)); (T_{zx}): average horizontal shear force per linear meter; (a): half of the beam depth (See **Fig.3**); ($Z_{\|}$): reference design value, parallel to the grain. (See **Table 8**); (e): spacing between mechanical connectors, horizontally arranged.

Table 14. According to the $\tau_{zx\ maximum}$ stresses, by [20].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q=\frac{(q)(L^*)}{2}$ [kN] (L^*=3.81m)	$(\tau_{\text{zx joint}})$ [kN/m^2]	$\left(\frac{\tau_{\text{zx joint}}}{2}\right)$ [kN/m^2]	$T_{zx}=\left(\frac{\tau_{\text{zx joint}}}{2}\right)(a)$ [kN/m]	$Z \|$ [kN]	Spacing $e=\left(\frac{Z \|}{T_{zx}}\right)$ [m]
Single	1.35	4.90	9.34	116.23	58.12	5.35	7.10	1.33
Steel	1.66	6.04	11.50	143.11	71.56	6.59	7.10	1.08
Plate	2.29	8.31	15.83	196.87	98.44	9.06	7.10	0.78
Double	3.12	11.33	21.58	268.43	134.22	12.36	7.10	0.57
Steel	3.75	13.59	25.90	322.19	161.10	14.83	7.10	0.48
Plate	4.37	15.86	30.22	375.95	187.98	17.31	7.10	0.41

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load (See **Table 5** and **Fig. 10**); (L^*): span between supports (See **Table 5** and **Fig. 10**); (Q): vertical shear at the support, for a simply supported beam (See **Fig. 7**, case I); ($\tau_{\text{zx max}}$): maximum horizontal shear stresses, at a point of coordinates ($x=b/2$; $y=a$), (See Eq. (5)); (T_{zx}): average horizontal shear force per linear meter; (a): half of the beam depth (See **Fig.3**); ($Z_{\|}$): reference design value, parallel to the grain. (See **Table 8**); (e): spacing between mechanical connectors, horizontally arranged.

Table 15. Rational Method based on [17], by [16].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	$Q(k)$	$(\tau_{\text{zx joint}})$ [kN/m^2]	$Z \perp$ [kN]	Spacing [e] $=\left(\frac{Z\ \perp}{(q)(k)}\right)$ [m]
Single	1.35	4.90	0.69	3.38	3.95	1.17
Steel	1.66	6.04	0.75	4.53	3.95	0.87
Plate	2.29	8.31	0.82	6.81	3.95	0.56
Double	3.12	11.33	0.80	9.06	3.95	0.44
Steel	3.75	13.59	0.83	11.28	3.95	0.35
Plate	4.37	15.86	0.86	13.64	3.95	0.29

Note: (b/a): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (a): half of the beam depth (See **Fig.3** and **9**); (b): half of the beam width (See **Fig.3** and **9**); (q): allowable uniform load. (See **Table 5** and **Fig. 10**); (k): coefficient of ratio between the resistant modulus of the steel cross-section and the total cross-section (See **Table 4**); ($Z_{\perp}$): reference design value, perpendicular to the grain. (See **Table 7**); (e):

spacing between mechanical connectors, horizontally arranged.

Table 16. Rational Method based on [17], by [19].

Composite Beam Case	$\left(\frac{b}{a}\right)$	(q) [kN/m]	(k)	$(q)(k)$ [kN/m]	$Z \perp$ [kN]	Spacing [e] $= \left(\frac{Z \perp}{(q)(k)}\right)$
Single	1.35	4.90	0.50	2.45	1.41	0.57
Steel	1.66	6.04	0.50	3.02	1.41	0.47
Plate	2.29	8.31	0.50	4.15	1.41	0.34
Double	3.12	11.33	0.50	5.66	1.41	0.25
Steel	3.75	13.59	0.50	6.80	1.41	0.21
Plate	4.37	15.86	0.50	7.91	1.41	0.18

Note: (*b*/*a*): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (*a*): half of the beam depth (See **Fig.3** and **9**); (*b*): half of the beam width (See **Fig.3** and **9**); (*q*): allowable uniform load. (See **Table 5** and **Fig. 10**); (*k*): coefficient of ratio between the resistant modulus of the steel cross-section and the total cross-section (See **Table 4**); ($Z_{\perp}$): reference design value, perpendicular to the grain. (For single shear failure mechanism. See **Fig.11** and T**able 11** B from [17], $Z_{\perp} = 310lbs \approx 1.41kN$); (*e*): spacing between mechanical connectors, horizontally arranged.

Table 17. Comparison of the spacing of horizontally arranged mechanical connectors obtained by the Rational Method, based on [17], and the Elastic Methods based on [20, 21, 22]

Case of Composite Beam Reinforced with Steel Plates	Steel Plate Thickness $[t_s]$	Ratio $\left(\frac{b}{a}\right)$	Spacing of the mechanical connectors, horizontally arranged "e" [m]						
			Elastic Theories					Rational Method based on NDS 2005	
			Reissner ($\tau_{zx\ joint}$) Eq. (2) by	Sadd ($\tau_{zx\ joint}$) Eq.(3) by	Berni ($\tau_{zx\ average}$) Eq. (4) by	Berni ($\tau_{zx\ joint}$) Eq. (6) by	Berni ($\tau_{zx\ maximum}$)	DeStefano [16]	Aghayere-Vigil [19]
Single	9.52	1.35	3.59	2.95	2.00	1.56	1.33	1.17	0.57
Steel	12.70	1.66	2.16	2.05	1.62	1.44	1.08	0.87	0.47
Plate	19.05	2.29	1.50	1.27	1.18	1.32	0.78	0.56	0.34
Double	12.70	3.12	0.98	0.80	0.86	1.25	0.57	0.44	0.25
Steel	15.87	3.75	0.77	0.62	0.72	1.22	0.48	0.35	0.21
Plate	19.05	4.37	0.64	0.51	0.62	1.20	0.41	0.29	0.18

Table 18. Determination of the required number of bolts in the support, according the Rational Method by [16], based on [17].

$\left(\frac{b}{a}\right)$	Span "L" [m]	Load "q" [kN/m]	Reduction Factor (k)	$Q = \frac{(q.k)(L)}{2}$ [kN]	$Z \perp$ [kN]	$Nr\ \ Bolts = \left(\frac{Q}{Z_{\perp}}\right)$
1.35	3.81	4.90	0.69	6.44	3.95	2.00
1.66	3.81	6.04	0.75	8.23	3.95	3.00
2.29	3.81	8.31	0.82	12.98	3.95	4.00
3.12	3.81	11.33	0.80	17.27	3.95	5.00
3.75	3.81	13.59	0.83	21.49	3.95	6.00
4.37	3.81	15.86	0.86	25.98	3.95	7.00

Note: (*b*/*a*): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (*a*): half of the beam depth (See **Fig.3** and **9**); (*b*): half of the beam width (See **Fig.3** and **9**); (*L*): span between supports (See **Table 5** and **Fig. 10**); (*q*): allowable uniform load. (See **Table 5** and **Fig. 10**); (*k*): coefficient of ratio between the resistant modulus of the steel cross-section and the total cross-section (**Table 4**.); (*Q*): vertical shear at the support, for a simply supported beam (See **Fig. 7** case I); ($Z_{\perp}$): reference design value, perpendicular to the grain. (See **Table 7**); ($Q/Z_{\perp}$): number of bolts in the support.

Table 19. Determination of the required number of bolts in the support, according the Elastic Method by [20]

$\left(\frac{b}{a}\right)$	Span "L" [m]	Load "q" [kN/m]	$\tau_{zx\ average}$ [kN/m^2]	$T_{zx}^{*} = \left(\frac{\tau_{zx\ average}}{2}\right)(2a)(b)$ [kN]	$Z \parallel$ [kN]	$Nr\ Bolts = \left(\frac{T_{zx}^{*}}{Z_{\perp}}\right)$
1.35	3.81	4.90	77.49	0.44	7.1	≈1.00
1.66	3.81	6.04	95.41	0.67	7.1	≈1.00
2.29	3.81	8.31	131.25	1.27	7.1	≈1.00
3.12	3.81	11.33	178.95	2.37	7.1	≈1.00
3.75	3.81	13.59	214.79	3.41	7.1	≈1.00
4.37	3.81	15.86	250.63	4.64	7.1	≈2.00

Note: (*b*/*a*): coefficient of ratio (width/depth) of the complete, homogenized cross-section (See **Table 4**); (*L*): span between supports (See **Table 5** and **Fig. 10**); (*q*): allowable uniform load (See **Table 5** and **Fig.10**); ($\tau_{zx\ average}$): average horizontal shear stresses, at a point of coordinates (*x*=*b*/2; *y*=*a*). (See Eq. (4)); (T_{zx}^{*}): average horizontal shear force; (*a*): half of the beam depth (See **Fig.3** and **9**); (*b*): half of the beam width (See **Fig.3** and **9**); ($Z_{\parallel}$): reference design value, parallel to the grain. (See **Table 8**); (Nr Bolts): number of bolts in the support.

Table 20. Variation of vertical and horizontal shear stresses as a function of ratio (width/depth)

$\left(\frac{b}{a}\right)$	$\beta_1 = \frac{\tau_{zy\ maximum}}{\tau_{zx\ maximum}}$	$\beta_2 = \frac{\tau_{zy\ maximum}}{\tau_{zx\ average}}$
1.35	2.6337	3.9506
1.66	2.1419	3.1228
2.29	1.5526	2.3289
3.12	1.1396	1.7094
3.75	0.9481	1.4222
4.37	0.8136	1.2204

Note: (*b*/*a*): coefficient of ratio (width/depth) of the complete, homogenized cross-section. (See **Table 4**); (β_1): ratio between the maximum vertical shear stress given by Eq. (1) and the maximum horizontal shear stress given by Eq. (5); (β_2): ratio between the maximum vertical shear stress given by Eq. (1) and the average horizontal shear stress given by Eq. (4).

5 Results and Discussion

The load-deflection curves, corresponding to the bending tests on the types of steel-reinforced composite wood sections (Type A, B, C, D, and E), reached different load values, with Type A and E sections reaching the highest loads compared to the rest of the sections. However, for the recorded deflections less than or equal to L/360, all the sections showed a linear behavior, compatible with a perfectly elastic material. For this reason, the calculation hypothesis within the elastic field under the Allowed Stress Design (ASD) methodology would be valid, which includes the simplified equations of the new Elastic Method, given by [20]. See (**Fig. 13**).

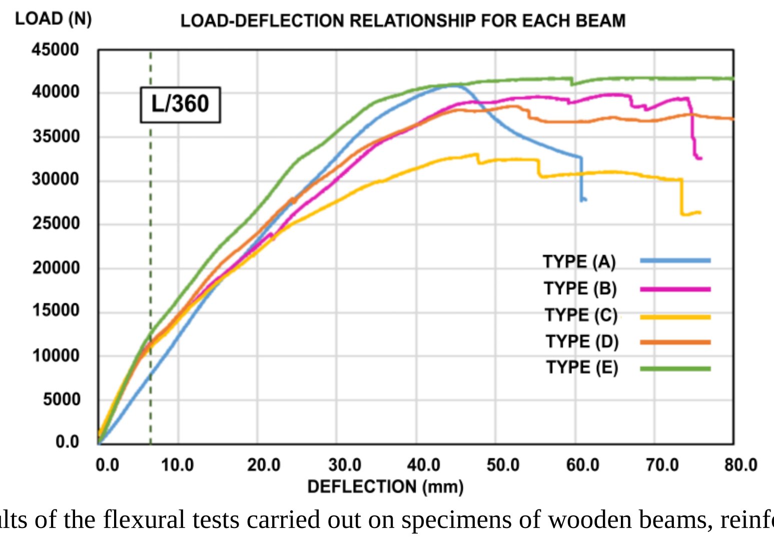

Fig. 13. The results of the flexural tests carried out on specimens of wooden beams, reinforced with steel rods and connectors such as bolts, nails, and a combination of both, showed a perfectly elastic behaviour of the material, within the *L*/360 range.

Fig. 14 shows the curves corresponding to the results obtained by applying the exact solution provided by the theory of elasticity according to [21, 22]. On the other hand, the results obtained according to the new Elastic Method, which uses the simplified equations of [20], and the Rational Method, in the variants [16] and [19], have also been represented.

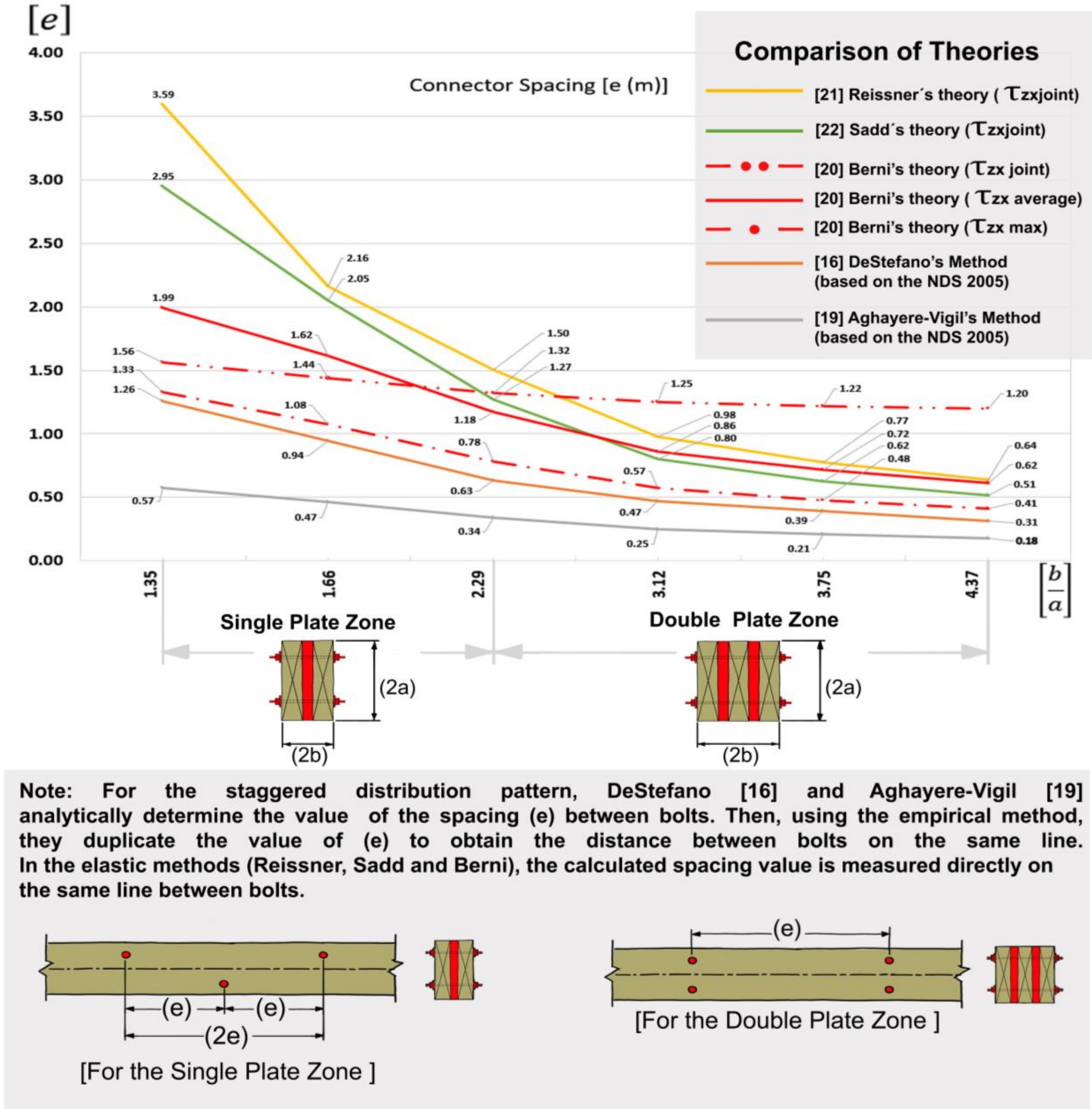

Fig. 14. Distance between mechanical connectors (bolts), according to the Rational Method, in its two variants [16], [19] and the simplified and original method given by [20] and the exact and complex solution provided by the Elasticity Theory according to the equation of [21, 22].

Conservative results were observed by applying the Rational Method. In **Fig. 14**, for the "single plate zone", there was a large disparity in the results between the Elastic Methods and the Rational Method. For example, even within the same Rational Method, large differences were recorded, depending on the version used, either by [16] or by [19].

Concerning the exact solutions provided by the theory of elasticity, an acceptable difference was recorded between the versions of [21] and [22]. On the other hand, according to the new Elastic Method, based on the equations given by [20], the equation corresponding to the mean value of the horizontal shear stresses was the one that obtained the best results, whose values were within the upper range, given by the exact solutions, and the lower range, given by the Rational Method.

However, within the "double plate zone", as the width/depth ratio increased, all the curves showed a similar trend and a narrowing of the margin of difference between the results. In that sense, the horizontal mean shear stress curve, corresponding to the new Elastic Method, given by [20], showed an excellent behavior, closer to the exact solution.

Within the calculation scenario, in all methods, the environmental conditions of humidity and temperature, included in the American standard according to [17], have been considered.

On the other hand, after the comparison of all methods, in **Fig. 15**, the analysis has been focused on the curves obtained by the Rational Method according to [16], the elastic curve according to [21] and the elastic curve obtained with the simplified average horizontal shear equation, given by [20]. For a given section with width/depth ratio, the corresponding ordinate represents the value of the spacing of the horizontally arranged mechanical connectors, according to the method considered. An adjustment factor, denoted Φ, has been defined, which allows for to comparison of the ordinates quickly and simply and to observe the differences between the calculation methods. The new Elastic Method demonstrated excellent behavior within the lower and upper limits. However, it also showed that the Rational Method yields conservative results for the spacing of the mechanical connectors.

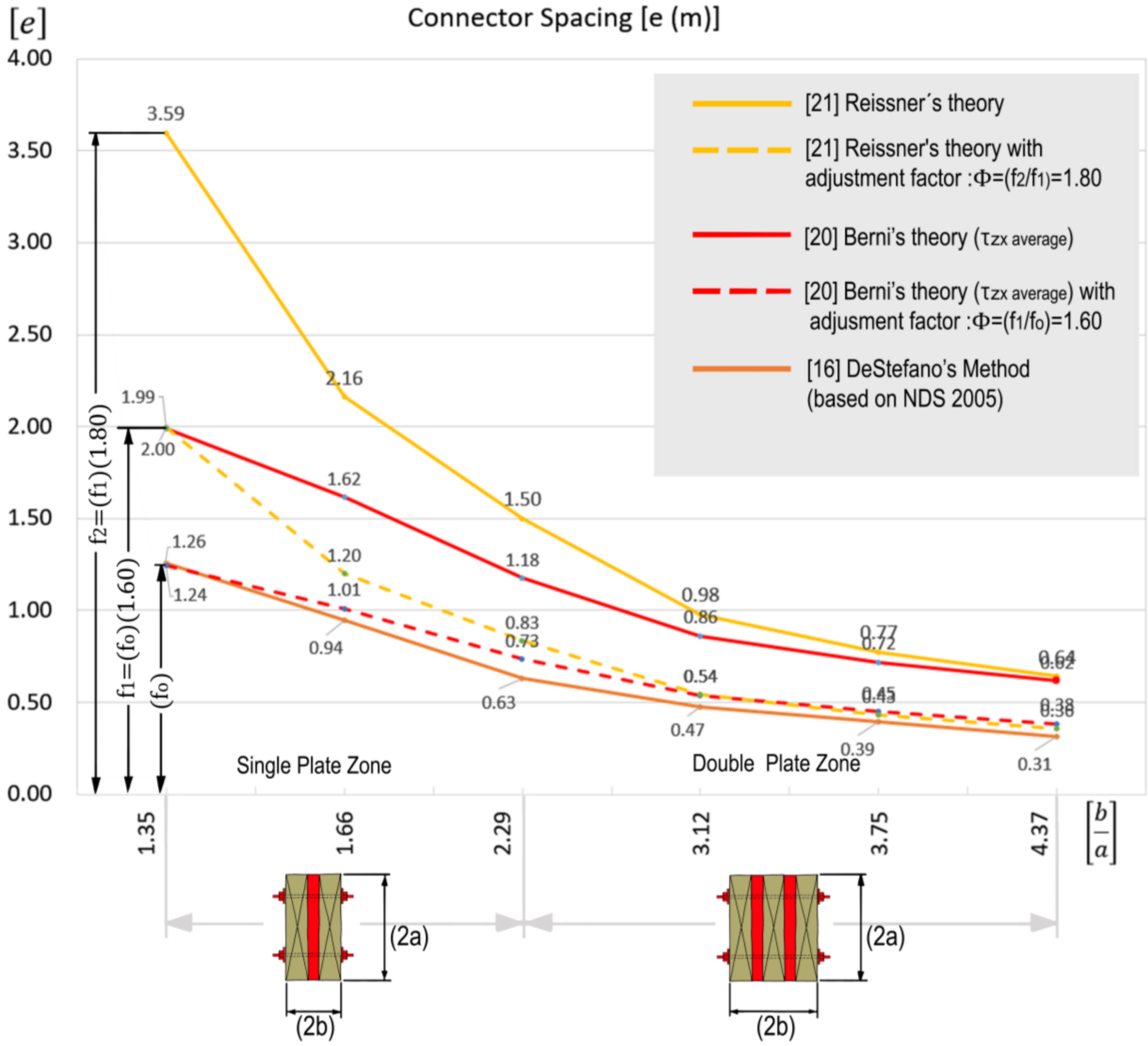

Fig. 15. The graph shows the calibration of some factors, about the Rational Method, in the variant given by [16], based on [17], to show the differences recorded concerning the elastic theories from [20] and [21].

Plotting the variation of vertical and horizontal shear stresses, through a dimensionless factor, denoted as 'β', a similar trend was observed in the plotted curves. As the width/depth ratio of the composite section increased, there was an increase in the horizontal shear stresses with respect to the vertical shear stresses. In other words, in beams of considerable width, relative to their depth, this situation naturally leads to the regulation of the spacing between the horizontal mechanical connectors. For example, for a width/depth ratio of 4.37, the horizontal shear stress is higher than the vertical shear stress and therefore requires a narrower spacing between mechanical connectors to absorb the higher shear stresses. Conversely, in sections with a width/depth ratio, as might result in 1.35, it is observed that the vertical shear stresses are higher than the horizontal shear stresses. This means that it is a slimmer section than the previous one, and the horizontal shear stress, being smaller, allows a larger spacing of the horizontal mechanical connectors. This situation can be seen in **Fig. 16** and **Table 20**.

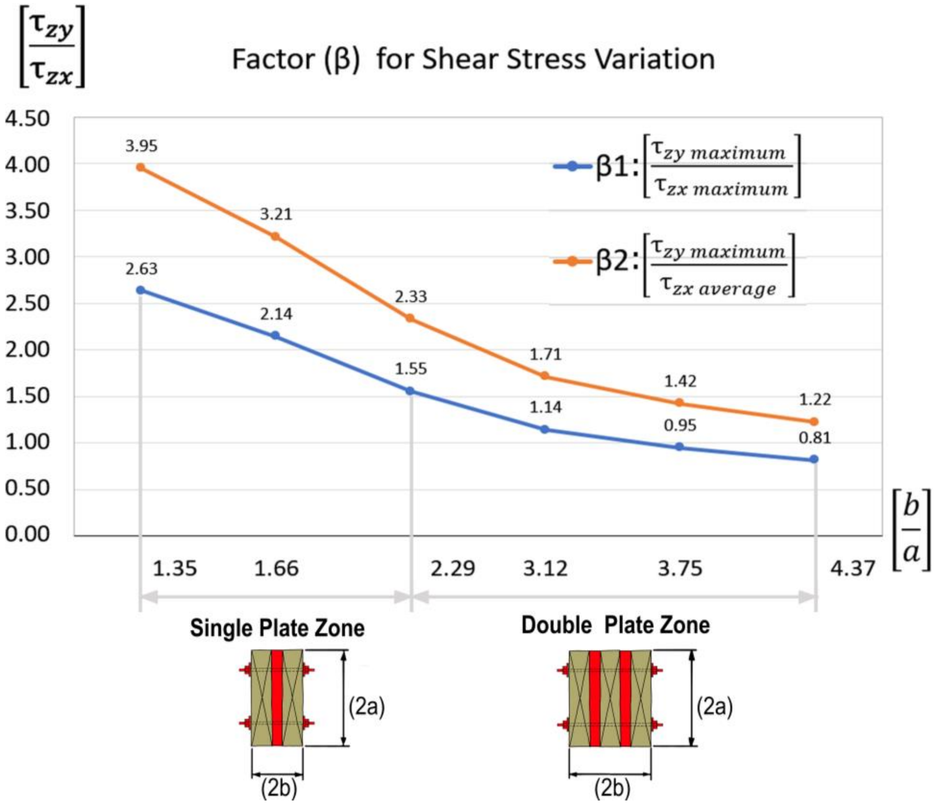

Fig. 16. The curves indicate that an increase in the width/depth ratio of the sections is accompanied by an increase in the horizontal shear stress.(Variation curves of factor β_1 and β_2, calculated in Table 20)

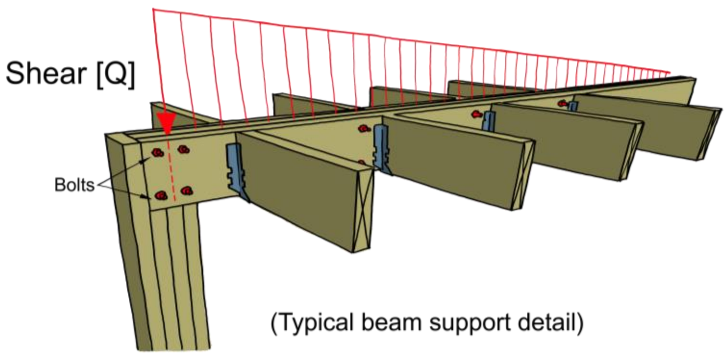

Fig. 17. The vertical shear force or reaction at the beam support is given for the Rational and Elastic methods, although there are substantial differences in the results according to Fig. 13.

Tables 18, show the difference between the classical methods, the so-called Rational Method, according to the version of [16], based on the American standard, and the new Elastic Method given by [20] and verified with the exact solution of the theory of elasticity. This big difference translates into the number of mechanical connectors in the support, as depicted in **Fig. 18**.

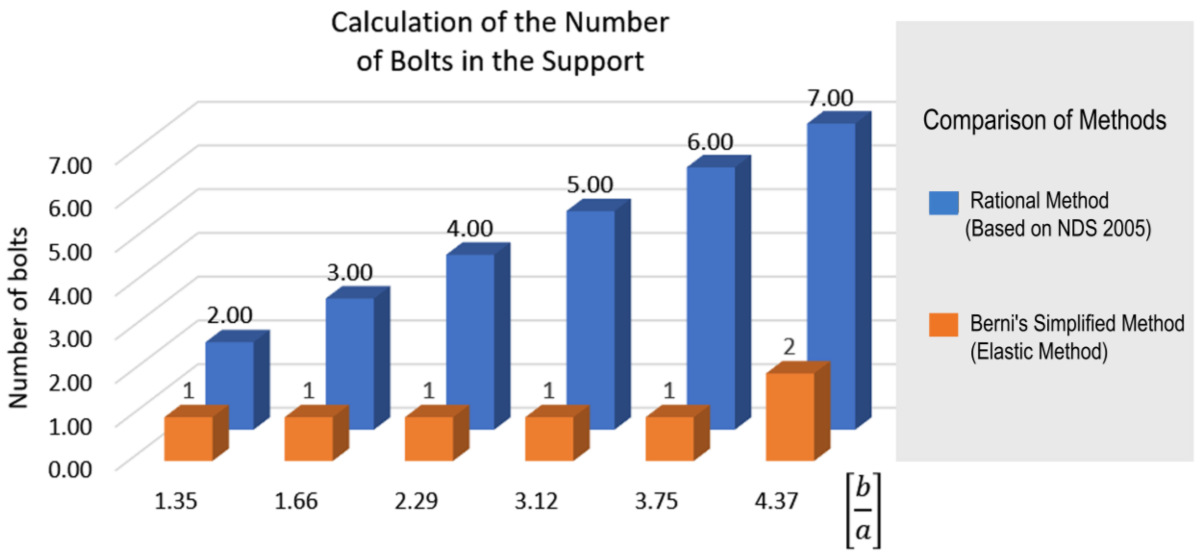

Fig. 18. For a simply supported beam, with a distributed load, with a free span of 3.81m, and varying the width/depth ratio, it is possible to visualize the big difference considering the method of [16] based on [17], and the elastic theories, in the version given by [20].

The Rational Method uses the double shear failure mechanism in mechanical connectors for flitch beams with single plates. Although it has been mentioned that flitch beams with double plates correspond to multiple shear, the American standard, perhaps with a simplistic criterion, indicates that they are double shear. On the other hand, the new Elastic Method, because it uses the law of horizontal shear stresses, takes into consideration more realistically the mechanism of failure by shear in mechanical connectors, being single, double, or multiple shears. However, to make a fairer comparison with the Rational Method, the same failure mechanism has been used. Despite this, substantial differences have been found in the number of mechanical connectors in the support.

6. Conclusions

1) The new Elastic Method allows the spacing of horizontally arranged mechanical connectors in composite cross-section beams, such as flitch and built-up beams, to be obtained with acceptable accuracy, simplicity, and considerable savings in calculation time.

2) To know the distribution of mechanical connectors accurately, the equation of the law of horizontal shear stresses (τ_{zx}) that develop between the contact planes between the vertical layers of materials is necessary. In this way, the spacing of the mechanical connectors can be calculated, taking into account the variation of the shear stress in the beam. This leads to an increase in the spacing and a reduction in the number of mechanical connectors, without impairing the strength of the cross-section.

3) Through bending tests, carried out on numerous specimens of composite timber beams, reinforced with steel and linked through mechanical connectors, the load-deflection curves demonstrated a perfectly elastic behavior of the composite section, provided that the deflection remained less than or equal to L/360.

4) Through bending tests on steel-reinforced composite wood beam specimens and the novel application of the exact solution provided by the theory of elasticity, first presented in 1946, the mechanical connectors' spacing results, obtained through the new Elastic Method, were successfully validated.

5) It was shown that, although the application of the Rational Method has led to positive experiences, they have been extremely conservative, as the spacing of the mechanical connectors does not take into consideration the true nature of the phenomenon; the horizontal shear stresses (τ_{zx}), which develop between the vertical contact planes between the layers of materials.

6) The Rational Method was applied, considering the hypothesis of single-shear and double-shear failure mechanisms, which led to substantial differences, even within the same method. In addition, the

Rational Method considers that a mechanical connector subjected to the multiple shear failure mechanism should be assimilated to the double shear case, which is considered in the American standard. However, in cases such as the composite section with a double steel plate, the mechanical connector works in multiple shear, so it is possible to consider the situation through the simplified Elastic Method.

7) The new Elastic Method offers the possibility to unify the analysis and design criteria in composite structures, particularly flitch and built-up beams, based on stress analysis instead of rules of thumb like the empirical method and eventually the Rational Method, which retains in its structure a remnant of the empirical method to establish the distribution pattern of the mechanical connectors.

8) The new method is an excellent alternative to the use of a finite element model, the implementation of which is not always accessible to all structural designers, depending on the complexity of the model.

9) Because the new Elastic Method is based on a simplified equation, which allows for to analysis of the case of stresses in wide beams up to 10 times their depth, in which case the horizontal shear stress (τ_{zx}) can equal and even greatly exceed the vertical shear stress (τ_{zy}). This allows to the analysis of a wide universe of configurations for composite beams of rectangular cross-section.

10) The Rational Method showed severe limitations, as its application requires a simply supported beam model subjected to uniform loading. An artifice was included in the manuscript that allows the Rational Method to be extended and applied to other states of different loads, giving the Rational Method greater generality.

11) The Rational Method allows for obtaining only a uniform spacing for the mechanical connectors, because it uses a uniform load for its determination. On the other hand, the new Elastic Method not only allows to vary the spacing of the mechanical connectors at the will of the structural designer, but also allows to design of the same distribution pattern of the mechanical connectors, which is adopted from the Empirical Method guidelines, in the structure of the Rational Method. Because the new method uses a shear stress derived from the horizontal shear stresses (τ_{zx}), it makes it possible to apply the new Elastic Method without any restrictions to any static or hyperstatic beam model, with any state of loading.

12) The American NDS 2005 standard uses two design philosophies: the Allowable Stress Design (ASD) method and the Load Resistance Factor Design (LRFD) method. Currently, only the classical Euler-Bernoulli bending equation and the Zhuravsky-Collignon equation are included in the ASD design method. Therefore, an equation to determine the horizontal shear stresses (τ_{zx}) is not included and could be considered in the ASD procedure.

13) A future line of research would be to develop a variant of the new Elastic Method, oriented towards the limit state method, which would make the method more general and comprehensive, as the Eurocode and the National Building Code of Canada (NBCC) use a design approach based on limit states, which consists of providing an adequate margin of resistance against certain limit states, resistance and serviceability. The limit states of resistance refer to the maximum load-bearing capacity of the structure. Serviceability limit states, on the other hand, are those that restrict the normal use and occupancy of the structure due to excessive deflections or vibrations to such an extent that they can cause severe damage to partitions, doors and windows, as well as discomfort and intense feelings of instability in the users of the building.

14) During the comparative analysis of the rational and Elastic Methods, environmental conditions such as humidity and temperature were considered using design factors from the American standard NDS 2005, which correspond to typical values for structures. A possibility to extend the research work would be to introduce several situations with different temperature and humidity variations and investigate the influence on the design parameters.

Conflicts of Interest

The author declares no conflict of interest.

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